Flexible Helices for Nonlinear Metamaterials

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After a decade of fruitful development, nonlinear metamaterials have received a new momentum with the introduction of alternative sources of nonlinearity, exploring a link between electromagnetism and other branches of physics, making use of thermal tuning[1,2] or mechanical coupling.[3] An interesting opportunity to create nonlinearity which combines such effects, is offered by a chiral particle, a well-known helical spiral,[4] which became popular in metamaterials design,[5] in accordance with the growing interest in metamaterial chirality.[6-11]

The helix is a very attractive “meta-atom” to bring an optomechanical analogy to electromagnetic metamaterials: it demonstrates a unique duality being, at once, an electromagnetic resonator and a mechanical spring. An interplay between the two responses of different types is provided through the dependence of resonance parameters on the spring geometry, while the sensitivity to heat is expressed through thermal expansion and temperature-dependent resistance. With an increasing incident power, the spring undergoes compression forced by the attracting currents in the neighbouring windings, while the growing temperature leads to an overall increase in size. Both effects act in the same direction, changing the geometrical conformation and shifting the electromagnetic resonance down to a lower frequency. This provides a nonlinear feedback making self-tuning processes possible, and promises an interesting range of nonlinear phenomena,[12] including a highly desirable reconfigurable chirality.[13,14]

Previous theoretical analysis[12] reached the conclusion that a fairly thin wire (with the wire radius being one hundred times smaller than the radius of the spring) is required in order to observe mechanical conformational changes. Accordingly, a preliminary experimental assessment of a single resonator[12] resulted in a system where thermal effect dominates, significantly limiting the range of nonlinear processes.

In this paper, we report an advanced design which enhances conformational self-tuning, making the latter stronger than the thermal contribution, and dramatically increasing the nonlinear response. With the improved manufacturing procedures, we are able to fabricate a large number of nearly identical elements for creating bulk metamaterials in the form of a lattice of helical meta-atoms (Figure 1). Our results open up a road to exploit the effect of nonlinear chirality for polarization conversion, beam splitting and modulation, and nonlinear chiral negative refraction, as well as nonlinear chiral optomechanics.

To achieve this goal, we use compact multi-turn helices made of thin copper wire with closely spaced windings (Figure 2). High temperature annealing increases the mechanical stability of the helices and minimizes thermal effects, while the large number of turns enhances the mechanical response to electromagnetic excitation. Indeed, the helical element considered earlier[12] was taken to be just a two-turn element for the purpose of precise analytical description. But for multi-turn helices, we may expect a more efficient current-induced compression as the compressing force increases through the interaction of multiple wire turns. Furthermore, the increase in the number of turns makes the entire helix less sensitive to possible fabrication inaccuracy, particularly to the variation of the total length of the wire.

This expectation is confirmed with our experiments on a piece of metamaterial excited with magnetic field at various input power levels (Figure 3). With an increase of supplied power to nearly 1 W, the resonance was shifted by 24 MHz from the original value. For the experiment we have used 16 helices with practically identical resonant frequencies, and we arranged them into a two dimensional 4 × 4 lattice, with their axes parallel to each other and also parallel to the magnetic field created.
the expected rotation of the polarization plane, the nonlinear response will not change except for an attenuation due to the dissipation. Also, in two-dimensional arrays, the mutual orientation of the helices may also vary (e.g., all the axes being perpendicular to the array plane, or lying in that plane); such variation leads to an overall shift of the resonance frequency due to modified mutual interaction, however this has no influence on the nonlinear behavior.

To describe the observed effects theoretically, we develop an approximate semi-analytical model. In general, evaluating the electromagnetic response of a multi-turn helix analytically is not straightforward[15,16] and is typically performed for infinite helices[17,18] or short spirals.[19] However, it is possible to estimate the resonance frequency from the geometric parameters of the helix using an analogy to the simple circuit model used for two-turn planar spirals.[20] We assume the model below to be a suitable approximation when the number of windings \( N \) is not too large and the axial length of the helix is not larger than its diameter (while a much longer helix behaves as a delay line and should be analyzed with transmission line approaches).[17]

For short multi-turn helices, we have found (see Supporting Information) that the following functional form can be used to evaluate the resonance frequency:

\[
\omega_0 = \frac{c}{\pi r} \left( \frac{\cos^{-1}(\xi/2\omega)}{2(N-1)\psi(\ln(8/\alpha) - 2)} \right)^{1/2}
\]

where \( r \) is the radius of the helix; \( \xi \) and \( w \) are the helix pitch and wire radius, respectively, both normalized to \( r \); \( N \) is the number of turns, and the correction factor \( \psi \) is determined empirically with the help of numerical simulations. We have confirmed with direct numerical simulations (CST Microwave Studio) that this equation is accurate within the simulation precision.

Similarly to a two-turn helix,[12] the balance between the attractive force between the windings and the counter-acting spring stiffness is determined by the condition

\[
\frac{\mu_0 \Xi I^2}{2\xi} = \frac{Grw^4}{4} (\xi_0 - \xi)
\]

where \( G \) is the shear modulus, \( I \) is the amplitude of the current flowing through the windings (the equation takes into account the fact that the force is determined by the time-averaged current), and \( \Xi \) is the enhancement factor due to the interaction of multiple windings; for 9 turns, \( \Xi \approx \sum_{n=1}^{4} \frac{1}{2} \approx 2 \). When a frequency scan at variable power is adopted, and sufficient time is allowed for the frequency sweep, the helix should be at the equilibrium position while at resonance (whereas the frequency of the resonance changes depending on the power). Then the amplitude of the induced current \( I \) depending on the incident power \( P \) can be found from the impedance equation written at resonance

\[
\frac{1}{w} \sqrt{\frac{\omega_0^2 \mu_0}{2\sigma}} = \varepsilon (\omega_0, P)
\]

where \( \sigma \) denotes conductivity and \( \varepsilon \) the induced electrostatic force per turn, depending on the geometrical parameters and excitation conditions. Note that this equation features an
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Figure 4. Change of the helix pitch with power, recalculated from the experimental data on the resonance shift with power (for a lattice, these are shown in Figure 3). Blue circles represent the data obtained for a single resonator, and red squares for the lattice; the surrounding contours indicate the measurement errors. Green solid curve shows the corresponding dependence obtained with the Equation 4. The axis on the right indicates \((\gamma - 1)\) as a measure of chirality.

Implicit dependence on \(\xi\) via the resonance frequency of the helix \(\omega_0\). Combining the Equations 1–3, we conclude that the magneto-mechanical behavior of the helix is governed by the approximate equation

\[
\frac{\xi (\omega_0 - \xi)}{\cos^{-1}(\xi/2w)} = \frac{4\mu_0 G \pi r \Sigma Y P}{\omega^2 \sqrt{2(N - 1)} \psi (\ln(8/w) - 2)} \tag{4}
\]

where \(Y\) is a coefficient determining the helix coupling to the power source. In our setup, by assessing the data in Figure 4 against the general theoretical functional form of Equation 4, we found \(Y = 1.5 \times 10^3\). However, we should note that a number of factors complicate the actual response, such as essential inhomogeneity of the magnetic field in the plane of the loop antenna as well as along the axes of the resonators (although these two effects partially compensate each other), non-uniformity of contraction, and so on. Nevertheless, as we can see from Figure 4, this equation provides a satisfactory qualitative matching to the experimental results with \(Y\) serving as the only fitting parameter.

In Figure 4, we present the change in the helix pitch, recalculated from the experimental data based on the measured resonance frequencies, and compare it with our theoretical fit. Chiral properties of the helix are directly proportional to its pitch, and can be characterized\(^{[21]}\) with the normalized (dimensionless) ratio between the electric \(p\) and magnetic \(m\) dipole moments along its axis,

\[
\gamma = \frac{|P|}{|M|} = \frac{c \xi}{\omega_0 \tau} = \frac{\xi}{\pi} \frac{\lambda}{2\pi r} \tag{5}
\]

where \(\lambda\) is the wavelength and \(c\) is the light velocity. To illustrate the arising nonlinear chirality, we provide an auxiliary axis on the right of Figure 4 which shows the magnitude of \((\gamma - 1)\) for the data presented, taken at the initial frequency of the resonance in the array.

We note that implementation of the flexible helices is much easier at a larger scale, so the applications for radio and microwave and GHz frequencies are straightforward. For THz range, fabrication is more challenging, particularly because the helices would need to be made of rather thin wire, making them fragile.

This problem may be solved by using a direct-laser writing technology\(^{[5,8]}\) with less rigid materials and a subsequent thin metal coating\(^{[16,22]}\) of the resulting helices. This would provide a sufficient conductivity (thanks to a strong skin-effect) and flexibility, while keeping wire radius comparably large. In the optical range, where metals are not suitable as conductors, a similar scheme can be realized by using high-permittivity dielectrics.\(^{[23]}\)

We also note that, for the proof-of-principle experiments, we have studied an anisotropic array. For a quasi-isotropic performance, the same helices can be arranged with a 3D-checkerboard logic, having their axes aligned along the three dimensions, with a sequence of three different orientations periodically patterned in each direction. Finally, a deliberate distinction can be made between a chiral lattice and a non-chiral racemic mixture, with the latter retaining the same nonlinear response but having no chiral properties.

In summary, we have presented the first experimental demonstration of conformationally nonlinear chiral metamaterial with the response governed by structural changes in its elements. Such nonlinear chirality can lead to interesting consequences when the operating frequencies lie near the resonance, especially above the resonance where negative effective permeability can be achieved. As we have shown, the effect obtained with a modest 1 W power is sufficient to move the resonance significantly over the resonance width, promising an efficient power-induced switching between positive and negative permeability along with chirality change. This will lead to interesting phenomena for wave propagation inside large metamaterial samples which can be easily manufactured. We believe that our work provides an encouraging implementation of metamaterials with conformational nonlinearity and paves a road towards optomechanical metamaterials.

Experimental Section

The helical resonators were fabricated by shaping a copper wire with 90 \(\mu\)m diameter. Two adjacent wires were tightly wound around the metal rod with ca. 2 mm diameter and were exposed to a moderate annealing by heating the metal rod, with the annealing optimized to provide best stability to the helix shape. The auxiliary wire was then wound off and the main helix was cut to produce a required number of turns. As a result, nine turn helical resonators with 1.16 mm \(\times\) 0.01 mm radius and 194 \(\mu\)m \(\times\) 1 \(\mu\)m pitch were selected.

The measurements over a single resonator were performed using a loop antenna (inner loop radius 5.51 mm, wire radius 1 mm) producing a strong magnetic field parallel to the helix axis. An Agilent HP E8362C vector network analyzer was employed to generate signals in the GHz range and to measure the response. An Agilent HP 83020A amplifier was used for high-power measurements.

The measurements over a 4 \(\times\) 4 lattice were performed in a similar way but a larger loop antenna (inner loop radius 9.47 mm) was used. The array was positioned inside the antenna loop within the plane of the loop, with the resonators suspended horizontally on thin plastic wires (where the windings can slide with a minimal friction), with their axes being horizontal and parallel to the loop axis. For the array assembly, the resonance frequency was carefully measured to select the resonators with individual resonances matched within a 0.01% relative accuracy.

An additional factor which helps to minimize the thermal effects lies in the way the measurements were performed: to avoid excessive overheating, we did not use a pump-probe experiment, but performed a series of quick frequency scans at various power levels, making a simultaneous power calibration with the help of power divider and vector
network analyzer. On the one hand, such a dynamic regime raises certain complications as the characteristic mechanical response time of the helix,

\[ \tau_s = \frac{4\pi^2 N_0^2}{\omega^2} \sqrt{\frac{2D}{3G}} \]

becomes relevant: the helix contraction has to follow the frequency sweep. On the other hand, this scanning method preserves the local effect of power, as it is effectively delivered at the resonance frequency during each scan.

**Supporting Information**

Supporting Information is available from the Wiley Online Library or from the author.

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