Resonance properties of domain walls in ferromagnets with a weak exchange interaction

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The spin dynamics of a domain wall is studied in an infinite ferromagnetic chain with an easy-axis anisotropy as well as in a chain of finite size. The dependence of the intrinsic mode frequency of the domain wall on the exchange interaction is studied for all admissible values of the latter. It is shown that this dependence is considerably modified in the region of transition of the domain wall from collinear structure to canted form. © 1998 American Institute of Physics. [S1063-777X(98)00507-6]

In this communication, we report on the investigation of nonlinear dynamics of spin systems. Magnetic solitons were first studied in Kharkov more than 25 years ago, and Prof. A. M. Kosevich was one of the pioneers in this field. The authors are indebted to him for introducing them to this interesting field of physical research at different times.

Nonlinear excitations of magnetically ordered media (domain walls, magnetic solitons) have been studied extensively for traditional magnets both from theoretical and experimental points of view.1,2 As a rule, the theoretical studies of these objects are carried out by using differential equations in the longwave approximation. However, it has been reported in a number of recent publications3–5 that magnets with a weak exchange interaction (in which the exchange integral J becomes of the order of, or smaller than, the single-ion anisotropy constant β) undergo qualitative variations in structure and domain wall dynamics, and the results obtained from a longwave description of such systems become inapplicable.

This problem has attained significance in recent years following the synthesis of new magnetic materials which satisfy the condition J~β. Examples of such materials are the quasi-one-dimensional magnets (CH3)3NH[NiCl2H2O, (C9H7NH)NiCl3, 1.5H2O,6 and layered antiferromagnets with a ratio J/β~10−2, e.g., (CH5)n(NH3)2MnCl4, (CnH2n+1)2NH3)2MnCl4,7–11 and most of the high-temperature superconductors and their isostructural analogs. Significantly, it is possible to change in the above layered antiferromagnets the number n of organic molecules intercalating the magnetic layers, thus opening the possibility of experimental investigation of the dependence of the structure and dynamic properties of such magnets on the value of the exchange integral J.

Van den Broek and Zijlstra3 were the first to show that for comparable values of the exchange interaction constant and the anisotropy constant, a domain wall ‘‘collapses’’ into a collinear structure of the size of atomic spacing with parallel and antiparallel spin orientations. Stepanov and Yablonskii12 studied experimentally the resonance properties of layered antiferromagnets and observed an additional absorption band in the magnon spectral gap. The authors attributed this band to the emergence of an intrinsic mode in domain walls. Since such modes do not exist in the longwave description of a magnet, their emergence is associated with the discreetness of the magnetic medium and the transformation of domain walls to collinear form. Goncharuk et al.4 actually established theoretically the existence of an intrinsic mode in a domain wall for values of the exchange integral J below the critical value corresponding to collapse of the wall.

In the present work, we demonstrate the existence of an intrinsic mode in a noncollinear domain wall for exchange interaction exceeding the critical value, and describe the variation of this mode in the vicinity of the critical value of J. This question is of importance not only for the investigation of magnetically ordered media, but also for the general development of ‘‘nonlinear physics’’ where the interest has been shifting in recent years towards essentially discrete systems.

The magnetization dynamics is studied in the framework of Heisenberg’s classical one-dimensional discrete model for an easy-axis ferromagnet, i.e., by using the Landau–Lifshitz discrete equation without damping. The total energy of a spin chain can be represented in the form
The equation for the intrinsic mode frequency $\Omega$ of a domain wall on the discreteness parameter $\lambda = J/(\beta a^2)$ of an infinite ferromagnetic chain for collinear (curve 1) and canted (curve 2) forms of the DW is given by:

$$E = \sum_n \left[ -\frac{J}{a} S_n S_{n+1} - \frac{\beta}{2} (S_n e_z)^2 \right].$$

(1)

where $S_n$ is the spin of a lattice site ($|S_n|^2 = 1$), $J$ is the exchange interaction constant ($J > 0$ for a ferromagnet), and $\beta$ is the single-ion anisotropy constant ($\beta > 0$) for an easy-axis ferromagnet with the easy axis along the $e_z$-axis. In this case, the Landau–Lifshitz equation can be written in the form:

$$\frac{1}{\omega_0} \frac{dS_n}{dt} + \left( \frac{l_0}{a} \right)^2 [S_n, (S_{n+1} + S_{n-1})] + [S_n e_z] (S_n e_z) = 0,$$

(2)

where $\omega_0 = 2\beta \mu_0 S_0 / h$ is the frequency of a uniform ferromagnetic resonance ($\mu_0$ is the Bohr magneton, $S_0$ is the nominal magnetization, and $l_0 = \sqrt{J/\beta}$ is the characteristic scale of spatial inhomogeneity of magnetization in a domain wall (“magnetic length”). It is convenient to go over to the complex quantity $\Psi_n = S_n + i S_n^x$ (the classical analog of the magnon creation operator) and spin projection onto the $z$-axis ($S_n^z = m_n$). In this case, if we measure time in units of $\omega_0^{-1}$ and introduce the parameter $\lambda = (l_0/a)^2$, we can write Eq. (2) in the form:

$$i \frac{d\Psi_n}{dt} + \lambda (\Psi_n m_{n+1} - \Psi_n m_{n-1} + \Psi_n m_{n+1} - \Psi_n m_{n-1})$$

$$+ \Psi_n m_n = 0.$$

(3)

It was shown in Refs. 3 and 4 that this equation has a static solution for a collinear domain wall with the following spin orientation:

$$m_{n=1} = 1 (n \equiv 0), \quad m_{n=0} = -1 (n > 0)$$

(4)

(the domain wall is located between spins with numbers 0 and 1) for values of the parameter $\lambda$ smaller than the critical value $\lambda_g = 3/4$. Substituting into Eq. (3) the solution (4) and the function $\Psi_n$ in the form $\Psi_n = \Psi_n e^{i\Omega t}$, we can easily obtain the intrinsic mode $\Omega$ of the domain wall for $\lambda < \lambda_g$:

$$\Omega^2 = \frac{1}{6} \left[ 6 - 4\lambda - \lambda^2 - \sqrt{4 + 8\lambda + \lambda^2} \right].$$

(5)

Curve 1 in Fig. 1 shows the dependence $\Omega(\lambda)$. For $\lambda < \lambda_g$, we obtain $\Omega \approx 1 - \lambda / 2$, while $\Omega \approx (8/3\sqrt{3})(\lambda_g - \lambda)^{1/2}$ near the critical point.

Let us consider the possibility of the existence of an intrinsic mode in a domain wall for exchange interactions exceeding the critical value ($\lambda > \lambda_g$). In this range of values of $\lambda$, the collinear structure corresponding to the solution (4) becomes unstable and the spin distribution in the domain wall becomes noncollinear ($m_n \neq \pm 1$). Using the smallness of the parameter ($\lambda - \lambda_g$), we can find this distribution in the vicinity of the critical value $\lambda = \lambda_g$. For a noncollinear structure, it is convenient to represent Eq. (3) in terms of the components $S_n'$:

$$\frac{dS_n'}{dt} + \lambda [S_{n+1}^z m_n^z - S_n^z m_{n+1}^z + S_n^z m_{n-1}^z - m_{n+1}^z S_{n+1}^z + m_{n-1}^z S_{n-1}^z] + S_n^z m_n^z = 0,$$

(6)

$$\frac{dS_n^y}{dt} + \lambda [m_n^x S_n^y - m_{n+1}^x S_{n+1}^y + m_{n-1}^x S_{n-1}^y - S_n^x m_{n+1}^y + S_n^x m_{n-1}^y] + S_n^x m_n^z = 0,$$

(7)

where $m_n^z = [1 - (S_n^x)^2] [2 - (S_n^x)^2]^{1/2}$.

To begin with, let us determine the static configuration of the domain wall by putting $S_n' = 0$ for the sake of definiteness. It follows from symmetry consideration that $m_{n=0} = -m_{n=1}$ and $S_n' = S_0' = 0 (n > 0)$. In the main (linear) approximation, the system of equations (6) and (7) can be reduced to the system:

$$(1 - \lambda) S_n^x - \lambda S_n^z = 0, \quad n = 1,$$

$$(\lambda + 1) S_n^x - \lambda (S_n^{x+1} + S_n^{x-1}) = 0, \quad n > 1,$$

(8)

whose solution has the simple form:

$$S_n^x = \frac{A}{3^{\frac{n-1}{2}}}, \quad \lambda = \frac{3}{4}, \quad n \geqslant 1,$$

(9)

where the constant $A$ is determined from the next approximation in perturbation theory. We introduce the small parameter of expansion

$$\varepsilon = \lambda - \lambda_g$$

(10)

and present the approximate solution in the form

$$S_n^x = \frac{A}{3^{\frac{n-1}{2}}} + Z_n,$$

(11)

where $A = \sqrt{3} \varepsilon$ and $Z_n = \varepsilon^{3/2} \ll A$. Retaining terms of the order of $\varepsilon^{3/2}$ in the static equations (6) and (7), we obtain the following system of difference equations:

$$-\frac{4}{3} \varepsilon A + \frac{1}{4} Z_1 - \frac{3}{4} Z_1 + \frac{1}{3} A^3 = 0, \quad n = 1,$$

$$-\frac{4}{9} \varepsilon A + \frac{5}{2} Z_2 - \frac{3}{4} (Z_1 + Z_3) - \frac{8}{81} A^3 = 0, \quad n = 2,$$

(12)

... etc.

It can be shown easily that the solution of this system can be chosen in the form $Z_1 = Z_2 = 0$, $Z_n (n \geqslant 3) \neq 0$. This corresponds to the following expression for the constant $A$:

$$A = 2 \sqrt{\varepsilon}$$

(13)
(a different choice of the sequence \( Z_n \) leads only to an additional expansion of the approximate solution with a different value of the small parameter of the expansion.) The first terms in the sequence \( Z_n \) and the asymptotic form for large values of \( n \) are given by

\[
Z_1 = Z_2 = 0, \quad Z_3 = -\frac{32}{3^2} \varepsilon^{3/2} - 0.126 \varepsilon^{3/2},
\]

\[
Z_4 = -\frac{1432}{3^7} \varepsilon^{3/2} - 0.644 \varepsilon^{3/2}, \quad \ldots
\]

\[
\ldots Z_n \approx \frac{2n}{3^n} \varepsilon^{3/2}, \quad n \gg 1.
\]

Thus, in the main approximation in the small parameter \( \varepsilon \), the solution for the static configuration of a domain wall has the form

\[
S_n^{(0)} = 0, \quad S_n^{(0)} = S_{n-1}^{(0)} = \frac{2 \sqrt{E}}{3^n}, \quad n \gg 1.
\]

It can be easily verified that this solution satisfies the system of static equations in which the nonlinear terms (cubic in \( S_1^2, S_2^2, S_2^3, S_1^1 \)) are considered only in two equations for spins in the vicinity of the center of the domain wall, while all the remaining equations are linearized in spin deviations \( S_n^i \):

\[
\left( \frac{1}{4} - \varepsilon \right) S_1^i - \frac{3}{4} + \varepsilon \right) S_2^i + \frac{1}{4} (S_1^i)^3 + \frac{3}{8} (S_1^i)^2 S_2^i
\]

\[
- \frac{3}{8} (S_1^i)(S_2^i)^2 = 0, \quad n = 1,
\]

\[
\frac{5}{2} S_n^i - \frac{3}{4} (S_{n+1}^i + S_{n-1}^i) = 0, \quad n > 1.
\]

Solving the dynamic problem in the same approximation and with the same accuracy, we retain nonlinear terms only with numbers \( n = 0,1 \) in the dynamic equations (6) and (7), and linearize them subsequently in small corrections to the static solution (15):

\[
S_n^i = S_n^{(0)} + \sqrt{2} W_n(t), \quad S_n^i = \sqrt{2} V_n(t),
\]

where \( W_n, V_n \ll S_n^{(0)} \).

Substituting a solution in the form

\[
V_n = v_n \cos \Omega t, \quad W_n = w_n \sin \Omega t
\]

into the obtained system of equations, we arrive at the final form of the system of linear differential equations for \( v_n \) and \( w_n \):

\[
\frac{5}{2} w_n - \frac{3}{4} (w_{n+1} + w_{n-1}) \pm \Omega v_n = 0,
\]

\[
\frac{5}{2} v_n - \frac{3}{4} (v_{n+1} + v_{n-1}) \pm \Omega w_n = 0
\]

and

\[
\left( 1 - \frac{2}{3} \varepsilon \right) w_1 - \frac{3}{4} \left( 1 - \frac{14}{3} \varepsilon \right) w_0 - \frac{3}{4} \left( 1 + \frac{2}{3} \varepsilon \right) w_2
\]

\[
+ \Omega v_1 = 0,
\]

\[
\left( 1 - \frac{2}{3} \varepsilon \right) w_0 - \frac{3}{4} \left( 1 - \frac{14}{3} \varepsilon \right) w_1 - \frac{3}{4} \left( 1 + \frac{2}{3} \varepsilon \right) w_{-1}
\]

\[
- \Omega v_0 = 0,
\]

\[
\left( 1 - \frac{2}{3} \varepsilon \right) v_1 - \frac{3}{4} \left( 1 - \frac{2}{3} \varepsilon \right) v_0 - \frac{3}{4} \left( 1 - \frac{2}{3} \varepsilon \right) v_2
\]

\[
+ \Omega w_1 = 0,
\]

\[
\left( 1 - \frac{2}{3} \varepsilon \right) v_0 - \frac{3}{4} \left( 1 - \frac{2}{3} \varepsilon \right) v_1 - \frac{3}{4} \left( 1 - \frac{2}{3} \varepsilon \right) v_{-1}
\]

\[
- \Omega w_0 = 0,
\]

where the signs ‘‘plus’’ and ‘‘minus’’ in Eqs. (19) and (20) correspond to numbers \( n > 1 \) and \( n < 0 \), respectively, and Eqs. (21) describe the dynamics of spins with numbers \( n = 1 \) and 0 adjoining the center of the domain wall. The intrinsic mode of the domain wall localized near its center corresponds to the following solutions of Eqs. (19)–(21):

\[
w_n = A \exp \left[ -\xi_1(n-1) \right] + B \exp \left[ -\xi_2(n-1) \right], \quad n \gg 1,
\]

\[
v_n = A \exp \left[ -\xi_1(n-1) \right] - B \exp \left[ -\xi_2(n-1) \right], \quad n \gg 1,
\]

\[
w_n = C \exp (\xi_1 n) + D \exp (\xi_2 n), \quad n \leqslant 0,
\]

\[
v_n = -C \exp (\xi_1 n) + D \exp (\xi_2 n), \quad n \leqslant 0,
\]

where

\[
\exp (-\xi_{1,2}) = \frac{1}{3} \left[ 5 \pm 2 \Omega - 4 \left( 1 \pm \frac{5}{4} \Omega^2 \right)^{1/4} \right].
\]

Substituting the solutions (22) and (23) into the system of equations (21), we arrive at the final expression for the dependence of the intrinsic mode frequency on the discreteness parameter \( \lambda \) of the spin chain. In the main approximation in the small parameter \( \varepsilon \), this dependence has the form

\[
\Omega \approx \frac{32}{3 \sqrt{39}} \sqrt{\lambda - \lambda^{\ast}}.
\]

Segment 2 of the dependence \( \Omega(\lambda) \) in Fig. 2 shows this dependence. Segment 3 of the same dependence shows the asymptotic form of the frequency dependence of the intrinsic mode obtained numerically by Bogdan et al.\textsuperscript{13} for large values of the parameter \( \lambda \).

Thus, we have shown that the domain wall in an easy-axis ferromagnet has an intrinsic mode over the entire range of values of the discreteness parameter \( J/\beta \), and the frequency dependence of this mode changes sharply in the vicinity of the critical value of this parameter corresponding to a transition of the domain wall from collinear to canted structure.

Unfortunately, the domain wall dynamics in an infinite spin chain with exchange interaction exceeding the critical
FIG. 2. Eigenfrequency spectrum of a finite spin chain containing a DW.

value can be studied only in a narrow range of values of $J$ near the critical value by making a number of simplifying assumptions. Hence it should be interesting to study the spin dynamics of such a system by using a simplified model containing a finite number of spins. Since the width of a domain-wall for small values of the exchange integral is close to the atomic spacing and the value of spin deviation decreases rapidly with increasing distance from its center, the intrinsic dynamics of the wall is determined actually by a small number of spins near the center.

Let us consider a chain formed by four spins in the “domain wall” configuration. In other words, we shall assume that, for small values of the exchange integral, the spin system has a collinear structure of the type (4): $m_1 = m_2 = -1$, $m_0 = m_{-1} = 1$. In the collinear phase, the system of equations (3) can be split into four linear equations for solutions of the type $\Psi_n = \psi_n \exp(i\Omega n)$:

$$
\begin{align*}
(\Omega - 1) \psi_1 + \lambda (\psi_2 + \psi_0) &= 0, \\
(-\Omega - 1) \psi_0 + \lambda (\psi_1 + \psi_{-1}) &= 0, \\
(\Omega - 1 + \lambda) \psi_2 + \lambda \psi_1 &= 0, \\
(-\Omega - 1 - \lambda) \psi_{-1} + \lambda \psi_0 &= 0.
\end{align*}
$$

These equations describe the frequency spectrum of the given system with a finite number of degrees of freedom and its dependence on the discreteness parameter $\lambda$, as well as the critical value $\lambda_*$ at which the domain wall goes over from collinear to canted form. The eigenfrequency spectrum is symmetric in the sign of $\Omega$ and contains four values. The dependence $\Omega(\lambda)$ has the form

$$
\Omega^2 = \lambda^2 + \lambda + 1 \pm \lambda \sqrt{\lambda^2 + 6\lambda + 5},
$$

where the minus sign corresponds to the intrinsic mode of the domain wall. By putting its frequency equal to zero, we get the value of the critical parameter $\lambda_* = 1 / \sqrt{2} \approx 0.71$ which is quite close to the corresponding value of the parameter $\lambda_* = 0.75$ for an infinite chain. Curve 1 in Fig. 2 shows the dependence (26) for the intrinsic mode. This dependence is identical to the function (5) for an infinite spin chain. The plus sign in formula (26) corresponds to a nonlocalized mode with $\psi_2 = -1.6\psi_1$. In the limit of an infinite chain, this state passes into antiphase spin vibrations of the upper boundary of the spin wave spectrum. Curve 3 in Fig. 2 shows the dependence $\Omega(\lambda)$ for this mode.

In the region $\lambda > \lambda_*$, the problem for a four-spin complex is solved exactly (in contrast to an infinite chain) even for a domain wall of canted form. It can be shown that its static configuration is described by the following solutions of the system of equations (6) and (7):

$$
\begin{align*}
m_1 &= -m_0 = -\frac{1}{4} \left[ 1 \pm (1/2) \sqrt{1 + z/(2\lambda - 1)^2} \right], \\
S_1^z &= S_0^z = \left[ 1 \mp (1/2) \sqrt{1 + z/(2\lambda - 1)^2} \right], \\
m_2 &= -m_{-1} = -\frac{1}{4} \left[ 1 \pm (1/2) \sqrt{1 + z} \right], \\
S_2^z &= S_{-1}^z = \left[ 1 \mp \sqrt{1 + z} \right],
\end{align*}
$$

(27)

where

$$
z(\lambda) = \frac{4\lambda^2 (2\lambda^2 - 1) (2\lambda^2 - 4\lambda + 1)}{(2\lambda - 1)(4\lambda^2 - 2\lambda - 1)}. 
$$

In the above formulas, we must take the upper signs for the region $\lambda_* < \lambda < \lambda_1$, upper for $(n = 2, -1)$ and lower signs for $(n = 0, 1)$ in the interval $\lambda_1 < \lambda < \lambda_2$, and lower signs for all $n$ in the interval $\lambda_2 < \lambda < \lambda_+$, where $\lambda_1 = 0.84$ is the root of the equation $z(\lambda) + (2\lambda - 1)^2 = 1$, while $\lambda_2 = 1.24$ is the root of the equation $z(\lambda) + 1 = 0$. The points $\lambda_1$ and $\lambda_2$ are not critical points, and all dependences at these points are smooth. (At these points, the rotation angles for spins with $n = 1$ and $2$ pass through the value $\pi/4$.) At the second critical point $\lambda = \lambda_+ = 1 + \sqrt{2}$, the “domain wall” type configuration disappears, and all spins turn in a direction perpendicular to the easy axis. In this unstable configuration, the intrinsic mode frequency again becomes equal to zero and the spin complex goes over to the homogeneous ground state. However, the description of a domain wall in the framework of a four-spin complex becomes physically invalid for values of the parameter $\lambda$ close to $\lambda_+$.

In order to describe the transformation of a spectrum in the region $\lambda > \lambda_*$, we linearize the dynamic equations (6) and (7) in the vicinity of the static configuration (27) (for the system with a finite number of spins under consideration, we must put in Eqs. (6) and (7) $S_n^- = S_n^+ = m_n = 0$ for all $n \geq 3$ and $n \leq -2$):

$$
\begin{align*}
S_n^+ &= S_n^+(0) + W_n(t), \\
m_n^+ &= m_n^+(0) - S_n^+(0) W_n(t) m_n^+(0),
\end{align*}
$$

(29)

where the quantities $m_n^+(0)$ and $S_n^+(0)$ are defined by (27) and $W_n, V_n, S_n^+(0)$ and $S_n^+(0)$. As in the case of an infinite chain, we seek the solution of the linearized equations in the form

$$
W_n = w_n \sin \Omega t, \quad V_n = v_n \cos \Omega t
$$

and obtain a system of eight linear equations for the quantities $w_n$ and $v_n$. Putting the determinant of this system equal to zero, we arrive at the final equation for determining the dependence of frequencies $\Omega(\lambda)$ for modes in the canted phase of the domain wall.
Nontrivial solutions for $\Omega^2(\lambda)$ (with $\Omega \neq 0$) satisfy cubic equations with a complex dependence of the coefficients on the parameter $\lambda$. We shall not write this equation here since it is quite cumbersome. However, we calculated the asymptotic dependences $\Omega(\lambda)$ near the critical values $\lambda = \lambda_+$ and $\lambda = \lambda_+$, and numerically plotted these dependences in the entire admissible range of values of $\lambda$ (curves 2 and 4 in Fig. 2). The functions $\Omega(\lambda)$ have a root dependence near $\lambda = \lambda_+$. This is the bifurcation point for the high-frequency mode whose degeneracy is removed due to violation of symmetry in the domain wall. For the ‘‘intrinsic mode,’’ curve 2 in Fig. 2 is quite close to the corresponding dependence for an infinite chain: $\Omega^2 \approx 3.09(\lambda - \lambda_+)$, (For an infinite chain, it follows from (24) that $\Omega^2 \approx 2.92(\lambda - \lambda_+)$. ) For not too large values of $\lambda$, curve 2 in Fig. 2 is also quite close to the corresponding dependence 2 in Fig. 1.

Thus, it can be seen that the model of spin chains of finite length can correctly describe the dynamics of a domain wall in the region of its transition from collinear to canted configuration. This is also confirmed by the correctness of all assumptions and approximations used in the analysis of an infinite spin chain containing a domain wall.

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