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Magnetic domains in spinor Bose-Einstein condensates

Michal Matuszewski, Tristram J. Alexander, and Yuri S. Kivshar

Nonlinear Physics Center and ARC Center of Excellence for Quantum-Atom Optics, Research School of Physics and Engineering, Australian National University, Canberra ACT 0200, Australia
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We discuss the structure of spin-1 Bose-Einstein condensates in the presence of a homogenous magnetic field. We demonstrate that phase separation can occur in the ground state of antiferromagnetic (polar) condensates, while the spin components of ferromagnetic condensates are always miscible, so that no phase separation occurs in them. Our analysis predicts that this phenomenon takes place when the energy of the lowest homogenous state is a concave function of the magnetization. We propose a method for creating spin domains by adiabatic switching-on of a magnetic field. We also discuss the phenomena of dynamical instability and spin domain formation. © 2010 American Institute of Physics. [doi:10.1063/1.3499231]

I. INTRODUCTION

The analysis of the properties of spin domains and magnetic solitons is one of the major topics in the theory of crystalline magnetic structures. But recent development in the physics of cold gases have given birth to an exciting new field where spinor dynamics and magnetic domain formation are the key ingredients of a new and seemingly different physics which, however, borrows many important results and techniques from solid state physics. More specifically, the spin degree of freedom of spinor Bose-Einstein condensates (BECs) leads to a wealth of new phenomena that do not exist in single component (spin-frozen) condensates. New spin-induced dynamics such as spin waves, spin-mixing, and spin textures have all been predicted theoretically and observed experimentally. The observation of these spin-dependent phenomena became possible with the development of optical traps which trap all spin components, rather than just the low-magnetic-field seeking spin states of magnetic traps. The effect of an additional small non-zero magnetic field on the condensate in optical traps was studied even in the seminal theoretical and experimental works. In fact, the interplay of spin and a magnetic field has been at the heart of some of the most impressive spin BEC experiments, including demonstrations of spin domains and spin oscillations, and observations of spin textures and vortices.

A spin-1 BEC in a magnetic field is subject to the well-known Zeeman effect. At low fields the linear Zeeman effect dominates, and leads to a Larmor precession of the spin vector about the magnetic field at a constant rate, which is unaffected by spatial inhomogeneities in the condensate. At higher magnetic fields the quadratic Zeeman effect becomes important, and leads to much more dramatic effects in the condensate, such as coherent population exchange between spin components and the breakdown of the single-mode approximation (SMA), which assumes that all the spin components share the same spatial density and phase profile. Studies of the behavior of spin-1 condensates in magnetic fields began with the work of Stenger et al., where the existence of magnetic (spin) domains was predicted and observed in the ground state of a polar $^{23}$Na condensate subjected to a magnetic field gradient. At the same time, the ground states of both ferromagnetic and antiferromagnetic (polar) condensates in homogenous magnetic fields were found to be free of spin domains in the local density approximation. It was found later that the SMA failed for the ground state of a condensate confined in a harmonic trap even in a homogenous field. Nevertheless, the SMA is still used in studies of spinor condensates because of its simplicity and validity over a broad range of experimental situations, in particular when the condensate size is smaller than the spin healing length, which determines the minimum domain size. On the other hand, dynamical instability leading to the spontaneous formation of dynamic spin domains has been found to occur in large ferromagnetic condensates prepared in excited initial states, while no such phenomenon has been predicted or observed in antiferromagnetic condensates. Similar instabilities have been found in the transport of both types of spin-1 condensates in optical lattices. It seems, however, that spin domains are only to be found in antiferromagnetic condensates in the presence of inhomogeneous magnetic fields or trapping potentials.

In this paper, we begin by surveying the results of our recent work and discuss the structure of spin-1 Bose-Einstein condensates in the presence of a homogenous magnetic field. We show that the translational symmetry of a homogenous BEC breaks down spontaneously and phase separation occurs in magnetized polar condensates if the magnetic field is strong enough. An analogous phenomenon has been predicted and observed previously in binary condensates. In contrast, domain formation driven by inhomogeneous external potentials or magnetic field gradients may be referred to as potential separation in accord with the terminology of Ref. 24. Here, we show that, for a range of experimental conditions, it is energetically favorable for the system to consist of two separate phases composed of different stationary states. Finally, we demonstrate numerically that this phenomenon can be observed in a polar condensate trapped in a harmonic optical potential.
II. MODEL

We consider a dilute spin-1 BEC in a homogeneous magnetic field in the z-direction. The mean-field Hamiltonian of this system is given by

$$H = \int d\mathbf{r} \sum_{j=0,+} \left( \frac{-\hbar^2}{2M} \nabla \psi_j^* \nabla \psi_j + \frac{\alpha_0}{2} n |\psi_j|^2 + V(\mathbf{r}) |\psi_j|^2 \right) + H_a,$$

(1)

where $\psi_j$, $\psi_0$, $\psi_e$ are the wave functions of atoms in magnetic sub-levels $m=-1,0,+1$, $M$ is the atomic mass, $V(\mathbf{r})$ is an external potential and $n=\sum n_j=\sum |\psi_j|^2$ is the total density of atoms. The asymmetric part of the Hamiltonian is given by

$$H_a = \int d\mathbf{r} \left( \sum_{j=0,+} E_j n_j + \frac{c^2}{2} |\mathbf{F}|^2 \right),$$

(2)

where $E_j$ is the Zeeman energy shift for state $\psi_j$ and the spin density is

$$\mathbf{F} = (F_x, F_y, F_z) = (\psi^* \hat{F}_x \psi, \psi^* \hat{F}_y \psi, \psi^* \hat{F}_z \psi),$$

(3)

where $\hat{F}_x, \hat{F}_y, \hat{F}_z$ are the spin matrices and $\psi=(\psi^+, \psi^0, \psi^-)$. The nonlinear coefficients are given by $c_0=4\pi\hbar^2/(2a_2+\alpha_0)/3M$ and $c_2=4\pi\hbar^2(a_2-\alpha_0)/3M$, where $a_2$ is the $s$-wave scattering length for colliding atoms with total spin $S$. The total number of atoms and the total magnetization, given by

$$N = \int n d\mathbf{r},$$

(4)

and

$$\mathcal{M} = \int \mathbf{F} d\mathbf{r} = \int (n_+ - n_-) d\mathbf{r},$$

(5)

are conserved quantities. The Zeeman energy shift for each of the components, $E_j$ can be calculated using the Breit-Rabi formula

$$E_{\pm} = -\frac{1}{8} E_{\text{HFS}} (1 + 4\sqrt{1 \pm \alpha + \alpha^2}) = g_I \mu_B B,$$

$$E_0 = -\frac{1}{8} E_{\text{HFS}} (1 + 4\sqrt{1 + \alpha^2}),$$

(6)

where $E_{\text{HFS}}$ is the hyperfine energy splitting at zero magnetic field, $\alpha=(g_I+g_J)\mu_B B/E_{\text{HFS}}$, where $\mu_B$ is the Bohr magneton, $g_I$ and $g_J$ are the gyromagnetic ratios of electron and nucleus, and $B$ is the magnetic field strength. The linear part of the Zeeman effect gives rise to a net shift in the energy, so we can eliminate it using the transformation

$$H \rightarrow H + (N+\mathcal{M})E_+/2 + (N-\mathcal{M})E_-/2.$$

(7)

This transformation is equivalent to eliminating the Larmor precession of the spin vector about the $z$-axis.\textsuperscript{21} We thus consider only the effect of the quadratic Zeeman shift. For sufficiently weak magnetic field we can approximate it by

$$\delta E = (E_+ - E_0)/2 = \alpha^2 E_{\text{HFS}}/16,$$

which is always positive.

The asymmetric part of the Hamiltonian (2) can now be rewritten as

$$H_a = \int d\mathbf{r} \left( -\delta E n_0 + \frac{c^2}{2} |\mathbf{F}|^2 \right) = \int d\mathbf{r} \left( e(\mathbf{r}) - \frac{c^2}{2} |\mathbf{F}|^2 \right),$$

(8)

where the energy per atom $e(\mathbf{r})$ is given by\textsuperscript{11}

$$e = -\delta E n_0 + \frac{c^2}{2} |\mathbf{F}|^2 = -\delta E n_0 + \frac{c^2}{2} (|f_+|^2 + m^2),$$

(9)

We write the wave functions as $\psi_j = \sqrt{n_j} \phi_j \exp(\mathbf{i} \mathbf{r})$, where the relative densities are $\rho_j=n_j/n$. We have also introduced the relative phase $\theta=\theta_0 + \theta_1 - 2\theta_0$, spin per atom $\mathbf{f}=\mathbf{F}/n$, and magnetization per atom $m=f_z=\rho_+ - \rho_-$. The transverse spin component per atom is $|f_x|^2 + |f_y|^2$.

The Hamiltonian (1) leads to the Gross-Pitaevskii equations describing the mean-field dynamics of the system

$$i\hbar \frac{\partial \phi_j}{\partial t} = [\mathcal{L} + c_2(n_+ + n_0 - n_-)] \psi_j + c_2 \psi_0 \phi_j^*,$$

$$i\hbar \frac{\partial \phi_0}{\partial t} = [\mathcal{L} - \delta E + c_2(n_+ + n_-)] \psi_0 + 2c_2 \psi_e \phi_0^*,$$

(10)

where $\mathcal{L}$ is given by $\mathcal{L} = -\hbar^2 \nabla^2 / 2M + c_0 n + V(\mathbf{r})$.

By comparing the kinetic energy with the interaction energy, we can define a characteristic healing length $\xi = 2\pi n / \sqrt{2Mc_0n}$ and spin healing length $\xi_s = 2\pi n / \sqrt{2Mc_2n}$. These quantities give the length scales of spatial variations in the condensate profile induced by the spin-independent or spin-dependent interactions, respectively. Analogously, we define the magnetic healing length as $\xi_m = 2\pi n / \sqrt{2Mc_m E}$.

In real spinor condensates, the scattering lengths $\alpha_0$ and $\alpha_2$ have similar magnitudes. The spin-dependent interaction coefficient $c_2$ is therefore much smaller than its spin-independent counterpart $c_0$. For example, this ratio is about 1:30 in a $^{23}$Na condensate and 1:220 in a $^{87}$Rb condensate far from Feshbach resonances.\textsuperscript{27} As a result, excitations that change the total density require much more energy than those which keep $n(\mathbf{r})$ close to the ground state profile. Here we shall assume that the amount of energy present in the system is not sufficient to excite the high-energy modes, and treat the total atom density $n(\mathbf{r})$ as constant.

III. CONDENSATE WITHOUT A TRAPPING POTENTIAL

The ground states of spin-1 condensates in a homogeneous magnetic field have been studied in a number of previous papers.\textsuperscript{2,13,28} The most common procedure involves minimizing the energy functional with constraints on the number of atoms $N$ and the total magnetization $\mathcal{M}$. The resulting Lagrange multipliers $p$ and $q$ serve as parameters related to the quadratic Zeeman shift $\delta E$ and the magnetization $\mathcal{M}$. An alternative method, elaborated in Ref. 13, involves minimizing the energy functional in the parameter space of the physically relevant variables $B$ and $m$. Most previous studies have assumed, however, that the condensate remains homogenous and is well described by the single-mode approximation; in particular, that the spatial structure observed in Ref. 2 resulted from the applied magnetic field gradient, while the BEC was assumed to be described well by the homogenous model at each point in space (local den-
sity approximation). In Ref. 13, the breakdown of the single-mode approximation was demonstrated numerically for a condensate confined in a harmonic potential.

We correct the previous studies by showing that when the condensate size is larger than the spin healing length $\xi_s$, the translational symmetry is spontaneously broken and phase separation occurs in magnetized polar condensates if the magnetic field is strong enough. This phenomenon takes place when the energy of the spin state with the lowest energy is a concave function of $m$ for a given $\delta E$. On the other hand, the energy is always a convex function of $m$ for ferromagnetic condensates, so that no phase separation occurs in them. Note that phase separation has been previously predicted in binary condensates$^{23,24}$ and in ferromagnetic condensates at finite temperature.$^7$

We construct ground states of the condensate using homogeneous stationary solutions of the GP equations (10) of the form

$$\psi_j(\mathbf{r}, t) = \sqrt{n_j} \text{e}^{-i(\mu_j + \mu_0)t + \theta_j},$$

where $\mu_j = \gamma_j \hbar \ell$ is a constant and $\mu_+ + \mu_- = 2\mu_0$ owing to a phase matching condition. Following Ref. 22, we distinguish several types of stationary states. The states where only a single Zeeman component is populated ($n_j = 1$ for specified $j = \pm 0$, or $+$) are denoted by $\rho_{\pm \rho_0}$ and $\rho_{+}$, respectively. The state where $n_0 = 0$ but $n_-, n_+ \neq 0$ is the two-component (2C) state. The three-component states are classified according to the value of $\theta = \theta_+ + \theta_- - 2\theta_0$. The states with $\theta = 0$ are called phase-matched (PM) states, and the ones with $\theta = \pi$ are called anti-phase-matched (APM) states. For more details about this classification and the properties of the stationary states, see Ref. 22.

The two types of domain structures depicted in Fig. 1 are composed of two different stationary states connected by a shaded region where all three components are nonzero. These two domain states have the advantage that the perpendicular spin is nonzero only in the transition region; hence, their energy is relatively low in polar condensates. In fact, these are the only phase-separated states that can be the ground states of a homogenous condensate. Their energies per atom in the limit of infinite condensate size, so that the relatively small intermediate region can be neglected, are

$$e_{\rho_{\pm \rho_0}} = |\bar{m}| e_{\rho_{\pm}} + (1 - |\bar{m}|) e_{\rho_0},$$

$$e_{2C+\rho_0} = \frac{\bar{m}}{m_{2C}} e_{2C_{\bar{m}=m_{2C}}} + \left(1 - \frac{\bar{m}}{m_{2C}}\right) e_{\rho_0},$$

where $m = M/N$ is the average magnetization and the magnetization of the 2C component $m_{2C}$ is a free parameter that has to be optimized to obtain the lowest energy state.

The ground states can be determined by comparing the energies of the phase separated states with those of the homogenous solutions. The form of Eqs. (12) indicates that phase separation will occur when the energy of the lowest homogenous state is a concave function of magnetization. The results for both polar and ferromagnetic condensates are summarized in Table I. In the cases when no phase separation occurs, our results are in agreement with those obtained in Ref. 13. Note that we have assumed that the condensate size is much larger than $\xi_s$ and $\xi_E$. For small condensates, the results of Ref. 13 are correct. In the case of high magnetic field strength, one of the Zeeman sub-levels is essentially depleted$^{13}$ and the condensate becomes effectively two-component. The existence of the $\rho_{\pm \rho_0}$ phase in a polar condensate can then be understood in terms of the binary condensate model.$^{23,24}$ We note that the experiment reported in Ref. 30, performed under these conditions, can be viewed as a first confirmation of phase separation in a spin-1 BEC in a homogenous magnetic field. The ground state was not achieved, however, and a multiple domain structure was observed.

Figure 2 shows a phase diagram for polar condensates obtained both numerically and using the analytical formulas of Table I. The ground state profiles for a quasi-1D condensate were found numerically by solving the 1D version of Eqs. (10).$^{21}$

$$i\hbar \frac{\partial \psi_ \pm}{\partial t} = \left[ \tilde{\mathbf{C}} + \tilde{\mathbf{C}}(\bar{n}_\pm + \bar{n}_0 - \bar{n}_\mp) \right] \psi_ \pm + \tilde{\mathbf{C}} \tilde{\mathbf{S}} \bar{\psi}_ \mp \psi_ \pm,$$
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with verified using the numerical time evolution according to Eqs.

matching condition. The stability of the resulting states was

N triangles representing 2C, open circles 2C + +

dient flow method, 32, 33 which yields a state that minimizes

solutions were found numerically using the normalized gra-

conditions on

trapping frequency. We have imposed periodic boundary

us to reduce the problem to one spatial dimension. 27, 31 The

spin healing length, and the nonlinear energy scale is much

radius of the transverse trapping potential is smaller than the

transverse trap with frequency $c_{0}$ because of the finite value of

the ratio $c_{0}$.

Evident in Fig. 1c and 1d.

The results of the preceding section can be verified ex-

trapping in 23Na, with solid

-correspond to numerical data obtained for the parameters of 23Na, with solid

1

| $\tilde{m}$ |

0

0.5

1.0

$\delta E/c_{2}n$

0.5

1.0

$\rho_{0}$

FIG. 2. Ground state phase diagram of the polar condensate. The symbols

correspond to numerical data obtained for the parameters of 23Na, with solid

triangles representing 2C, open circles 2C + + and open squares $\rho_{+} + \rho_{0}$. The

solid lines and shading correspond to the analytical formulas from Table 1.

with

$\hbar \frac{\delta \tilde{\psi}_{0}}{\delta t} = [\tilde{L} - \delta E + \tilde{c}_{z}(\tilde{n}_{+} + \tilde{n}_{-})]\tilde{\psi}_{0} + 2\tilde{c}_{z}\tilde{\psi}_{+}\tilde{\psi}_{-}^{a}$, (13)

where

$\tilde{c}_{0} = 4\hbar \omega_{z}(2a_{z} + a_{0})/3$, $\tilde{c}_{z} = 4\hbar \omega_{z}(a_{z} - a_{0})/3$, $\int dx|x|\tilde{\psi}_{j} = N$, and $\omega_{z}$ is the transverse trapping frequency. We have imposed periodic boundary

conditions on $\tilde{\psi}_{j}(x)$ and used the parameters corresponding to a 23Na BEC containing $N=5.2 \times 10^{4}$ atoms confined in a transverse trap with frequency $\omega_{z} = 2\pi \cdot 10^{3}$. The Fermi radius of the transverse trapping potential is smaller than the spin healing length, and the nonlinear energy scale is much smaller than the transverse trap energy scale, which allows us to reduce the problem to one spatial dimension. 27, 31 The solutions were found numerically using the normalized gradient flow method, 32, 33 which yields a state that minimizes the total energy for given $N$ and $M$, and satisfies the phase matching condition. The stability of the resulting states was verified using the numerical time evolution according to Eqs. (13). The slight discrepancy between the numerical and analytical results can be explained by the finite size of the condensate (the box size was $\sim 10\xi$), and by the deviation from the assumption that the total density is constant (see the discussion at the end of Sec. II). Because of the finite value of the ratio $c_{z}/c_{0}$ there is a slight density modulation, as is evident in Fig. 1c and 1d.

IV. CONDENSATE TRAPPED IN A HARMONIC OPTICAL POTENTIAL

The results of the preceding section can be verified experimentally in configurations involving toroidal or square-

shapes optical traps. 34 In most experiments on BECs, how-

ever, harmonic potentials are used. The relevance of these results is not obvious in the case of harmonic trapping, since the coefficient $\delta E/(c_{2}n)$, one of the main parameters controlling the condensate properties, varies in space owing to the variable total density $n$.

The ground states in a highly elongated harmonic trap, where the parallel part of the potential has the form $V(x) = (1/2)M\omega_{x}^{2}x^{2}$, are shown in Fig. 3. 35 We can see that as the magnetic field strength is increased, phase separation occurs and the $\rho_{+} + \rho_{0}$ domain state is formed. However, in contrast to the previous case, the transition is not sharp, and in particular there is no distinct 2C+ $\rho_{0}$ phase for any value of the magnetic field. Note that the state in Fig. 3a is also spatially separated due to different Thomas-Fermi radii of the $\psi_{+}$ and $\psi_{-}$ components; however, this is an example of potential separation, rather than phase separation, 24 since it does not occur in the absence of the potential. On the other hand, Fig. 3d shows that the components of ferromagnetic condensate are miscible even in a strong magnetic field. In the regions where the wave functions overlap, the relative phase is $\theta = 0$ for ferromagnetic and $\theta = \pi$ for polar ground states, since these configurations minimize the spin energy (9). The characteristic feature of phase separation in a polar BEC is that the $m=0$ domain tends to be localized in the center of the trap, as shown in Figs. 1b and 1c. This can be explained by calculating the total asymmetric energy of the condensate (8), again assuming that the contribution from the intermediate region connecting the domains is negligible,

$$H_{\theta} = \int_{\rho_{0}}^{1} dr\nu(\delta E) + \int_{\rho_{+}}^{1} dr\nu c_{2}n$$

$$= -\delta E(N - |M|)$$

+ \frac{c_{2}N}{2}(\langle n \rangle_{\rho_{+}}$$

where $\langle n \rangle_{\rho_{+}}$ is the mean condensate density within the region of the $\rho_{+}$ domain. We see that the energy will be minimal if this domain is localized in the outer regions, where the condensate density is low.

V. GENERATION OF SPIN DOMAINS

We propose a method for creating the spin domains described in the previous section by adiabatically switching-on a magnetic field. We start with a condensate with all the atoms in the $m=1$ sublevel, in the ground state of a harmonic potential. Subsequently, some of the atoms are transferred to the $m=-1$ component during the rapid adiabatic transition process. The magnetic field is then suddenly switched off. In this way we can obtain a condensate with arbitrary magnetization in the 2C state, which is the ground state for $B=0$ (see Fig. 2). Next, we gradually increase the magnetic field strength in an adiabatic process according to the formula

$$B = \sqrt{\frac{t}{t_{\text{switch}}}} B_{\text{final}}$$

where $t=0$ at the time the switching process begins, $t_{\text{switch}}$ is the switching time, and $B_{\text{final}}$ is the desired final value of the magnetic field. The form of Eq. (15) ensures that the quadratic Zeeman splitting grows linearly in time. We have confirmed that this condition improves the adiabaticity of the generation process. Some examples of the evolution of the condensate are shown in Fig. 4. The left column shows the time dependence of the atom density in the initially unoccupied $m=0$ component, and the right column shows the final domain profiles. These should be compared with the ground state profiles in Fig. 3. Domains are created in both the low and high magnetic field cases over times on the order of seconds, as shown in panels (a, d) and (b, e). However, when the switching time is significantly reduced, the process is no
longer adiabatic, so that multiple metastable domains are formed, as can be seen in panels (c, f). This picture is in qualitative agreement with the experiment,\textsuperscript{30} where metastable spin domains were formed by a nonadiabatic process within 50–100 ms.

VI. SPIN DOMAINS AND DYNAMICAL STABILITY

The above results show that domain structure formation in polar condensates does not occur in ferromagnetic BECs. This may seem to contradict the common understanding of ferromagnetism and the results of the quenched BEC experiment in Ref. 9. The conventional picture of a ferromagnet involves many domains pointing in various directions separated by domain walls. A similar structure was observed in Ref. 9. These cases correspond, however, to situations in which there is an excess kinetic energy in the system owing to finite temperature or to excitation of spatial modes. On the other hand, our study is limited to the ground state at $T=0$. It is easy to see from Eq. (9) that in zero magnetic field the ground state of a ferromagnetic BEC will always consist of a single domain with maximum possible value of the spin vector $|\vec{S}|=1$, pointing in the same direction at all points in space. When the temperature is finite, however, more domains can be formed, each with a different direction of the spin vector.

We emphasize that the domain structure of the ground state in polar condensates is very different from the domains formed when kinetic energy is injected into the system (as in Ref. 9). The latter constantly appear and disappear in a random sequence.\textsuperscript{9,16,17,21,35,36} On the other hand, the ground state domains are stationary and are positioned in the center of the trap. They exist in the lowest energy state, while the dynamical domains require some amount of kinetic energy to be formed. Ground state domains can be prepared in an adiabatic process, involving adiabatic rf sweep or a slow change of the magnetic field,\textsuperscript{30,35} while the kinetic domains require a sudden quench.\textsuperscript{9,35}

The dynamical instability of ferromagnetic condensates that leads to spontaneous formation of spin domains has been investigated theoretically\textsuperscript{18,16,35} and observed experimentally.\textsuperscript{9} An analogous phenomenon has been predicted recently for polar condensates in a magnetic field.\textsuperscript{21}

Here we correct the results of Ref. 21, by noting that the $\rho_\parallel=1$ state is stable in ferromagnetic condensates for $\delta E > 2|c_2|n$ and the $2C$ ($\rho_\parallel=0$) state is stable in polar BECs if $\delta E < m_3/2$. These two states become the ground states for the corresponding ranges of the parameters. By investigating stability in various ranges of parameters, we can formulate a phenomenological law governing the dynamical stability of condensates: (i) The only stable state for both polar and ferromagnetic BECs in a finite magnetic field is the ground state, as shown in Table I; (ii) in zero magnetic field, the same is true for ferromagnetic condensates; but all stationary states of polar condensates are dynamically stable in zero magnetic field.\textsuperscript{16,18,21} The reason for the stability of polar condensates in the zero magnetic field case is not yet clear. We note that in a weak magnetic field polar condensates may
also be effectively stable on a finite time scale. As shown in Ref. 21, in this latter case the growth rate of the unstable modes is proportional to the fourth power of the magnetic field strength. The time required for development of the instability may be much longer than the condensate lifetime.3

VII. CONCLUSIONS

We have studied the ground state of a spin-1 BEC in the presence of a homogeneous magnetic field with and without an external trapping potential. We have found that without a trapping potential the translational symmetry can be spontaneously broken in a polar BEC, with formation of magnetic domains in the ground state. We have shown that these results may be used to understand the ground state structure in the presence of a trapping potential by mapping the locally varying density in the trap onto the homogenous state. We have found that, depending on the magnetic field, the antiferromagnetic BEC ground state in a trap displays pronounced spin domains within a range of possible experimental conditions. Finally, we have discussed the relationship between the phenomenon of phase separation and the dynamical instability leading to the formation of dynamic spin textures.

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E-mail: ysk@internode.on.net


2J. Stenger, S. Inouye, D. M. Stamper-Kurn, H.-J. Miesner, A. P. Chikkatur,