Spatiotemporal discrete surface solitons
in binary waveguide arrays

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Abstract: We study spatiotemporal solitons at the edge of a semi-infinite binary array of optical waveguides and, in particular, predict theoretically the existence of a novel type of surface soliton, the surface gap light bullets. We analyze the stability properties of these solitons in the framework of the continuous-discrete model of an array of two types of optical waveguides.

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OCIS codes: (190.4420) Nonlinear optics, transverse effects in; (190.5530) Nonlinear optics: Pulse propagation and solitons; (190.5940) Self-action effects.

References and links
1. Introduction

Surface modes are regarded as a special type of electromagnetic waves propagating along interfaces separating different dielectric media. Recently, the interest in the study of electromagnetic surface waves has been renewed after the theoretical prediction [1] and subsequent experimental demonstration [2] of nonlinearity-induced self-trapping of light near the edge of a one-dimensional waveguide array with self-focusing nonlinearity that can lead to the formation of a discrete surface soliton. A similar effect of light localization has been predicted theoretically and observed experimentally for defocusing nonlinear media [3, 4].

Recently, we have studied the spatiotemporal evolution of nonlinear surface waves and surface solitons and suggested an important concept of spatiotemporal surface solitons [5]. These solitons provide an important generalization of a wider class of optical spatiotemporal solitons [6], often referred to as light bullets in the three-dimensional case [7]. The study of the properties of light bullets attracted attention of many research groups as an unique opportunity to create self-supporting fully localized objects in both space and time. In particular, the existence and properties of continuous-discrete spatiotemporal solitons have been extensively investigated in cubic [8, 9, 10, 11] and quadratic [12, 13] nonlinear optical media.

In this work, we extend this analysis to the case of spatiotemporal solitons in binary waveguide arrays. In the tight-binding approximation, the binary waveguide array is known to support two different types of discrete optical solitons localized in the total-internal-reflection (TIR) gap and Bragg-reflection (BR) gaps, previously analyzed theoretically [14, 15, 16] and observed experimentally [17]. We consider two types of truncated binary waveguide arrays taking into account the spatiotemporal evolution of light near the edge of the waveguide array. We combine the key features of both continuous and discrete nonlinear models and analyze, for the first time to our knowledge, the existence and properties of continuous-discrete soliton families describing spatiotemporal discrete surface solitons in the binary arrays.

2. Model

We consider propagation and localization of light in a semi-infinite periodic binary array of alternating wide and narrow weakly coupled optical waveguides, as shown schematically in Figs. 1(a,b). As was established earlier [14, 15, 17] for infinite binary waveguide arrays, the properties of spatial discrete solitons can be effectively managed by controlling the geometry of the waveguide array, e.g. the parameters of two types of the waveguides.

![Schematic structure of a binary array of weakly coupled optical waveguides truncated at (a) wide and (b) narrow waveguides, respectively.](image-url)

Following the earlier analysis [14, 15], we describe the binary array of two kinds of the waveguides (A and B; wide and narrow) in the tight-binding approximation. The total field is...
decomposed into a superposition of weakly overlapping modes of the individual waveguides,

\[ E(x,y,z,t) = \sum_n E_n(z,t) e_n(x,y,\omega_n) \exp\left[ i (k_n^2 z - \omega_n t) \right] + c.c. \]

where \( z \) is normalized with respect to the coupling constant, and \( t \) is the time normalized with respect to the ratio of group velocity dispersion and coupling constant. \( E_n(z,t) \) are the slowly varying envelope and \( e_n(x,y,\omega_n) \) the mode profile in waveguide \( n \) at the center frequency of the pulse \( \omega_n \) of the pulse. The normalized propagation constant \( k_n^2 = k_0 + \rho_n \) contains a constant contribution \( k_0 \) and a part that accounts for the inhomogeneity of the array. We take into account the spatiotemporal evolution of light, similar to the earlier studies [8, 9, 10, 11], but also assume that our waveguide array is truncated so that the light localization occurs near the edge of the waveguide array (see Fig. 1). The corresponding rescaled equations for the mode amplitudes take the form (see also Refs. [14, 15]),

\[
i \frac{\partial E_n}{\partial z} - \gamma_n \frac{\partial^2 E_n}{\partial t^2} + (E_{n+1} + E_{n-1}) + (\rho_n + \sigma_n |E_n|^2)E_n = 0,
\]

where \( n = 0, 1, \ldots, E_{-1} \equiv 0 \) due to the structure termination. \( \gamma_n \) is the dispersion coefficient, \( \rho_n \) characterizes the linear propagation constant of the mode guided by the \( n \)-th waveguide. \( \sigma_n \) are the effective nonlinear coefficients. To simplify our analysis we neglect absorption and also consider identical nonlinear and dispersion coefficients.

For the structure shown in Fig. 1(a), i.e. for a binary array truncated at the wide waveguide, we put \( \rho_{2n} = \rho \) and \( \rho_{2n+1} = -\rho \), whereas for the structure shown in Fig. 1(b), i.e. for a binary array truncated at the narrow waveguide, we have \( \rho_{2n} = -\rho \) and \( \rho_{2n+1} = \rho \). We consider also \( \sigma = -\gamma = -\text{sign}(\rho) \). In the numerical simulations outlined below we take \( \rho = 0.6 \) and also consider both types of nonlinearity, i.e. \( \sigma = \pm 1 \).

### 3. Spatiotemporal surface solitons

First, we are looking for spatiotemporal soliton solutions of this nonlinear model in the form \( E_n(t; z) = \exp(i\beta z) E_n(t) \), where \( \beta \) is the nonlinearity-induced shift of the waveguide propagation constant, serving likewise as a family parameter, and the envelope \( E_n(t) \) describes the temporal evolution of the soliton-like pulse in the \( n \)-th waveguide. We find different solutions localized near the surface and decaying far away from it, and also localized in time; we solve the stationary version of Eqs. (1) by using a standard band-matrix algorithm [18] to deal with the corresponding two-point boundary-value problem.

Figures 2(a-d) show several examples of the nonlinear spatiotemporal continuous-discrete localized states (‘discrete surface light bullets’ or spatiotemporal discrete surface solitons) located near the edge of the binary array for the case of the focusing nonlinearity. Examples of unstaggered spatiotemporal solitons corresponding to points (a)-(d) in Fig. 3(a), found at \( \beta = 4 \).

To make a preliminary conclusion about the linear stability of each nonlinear surface state found numerically, we calculate the total mode power [see Fig. 3(a)],

\[
P(\beta) = \sum_{n=0}^{\infty} \int_{-\infty}^{+\infty} |E_n(\beta)|^2 dt,
\]

and also the second conserved quantity of the dynamical system (1), the system’s Hamiltonian,

\[
H = \sum_{n=0}^{\infty} \int_{-\infty}^{+\infty} \left\{ |E_n - E_{n-1}|^2 - (2 + \rho_n) |E_n|^2 - \gamma_n \left[ \frac{\partial E_n}{\partial t} \right]^2 - \frac{1}{2} \sigma_n |E_n|^4 \right\} dt,
\]

defined for the inverted spatial and temporal variables such that \( i(\partial E_n/\partial z) = -\delta H/\delta E_n^*, z \) playing a role of the evolution variable. We expect that the stable spatiotemporal solitons should
Fig. 2. Examples of unstaggered spatiotemporal solitons corresponding to points a-d in Fig. 3, for $\beta = 4$: (a) stable and (b) unstable discrete surface light bullets in the array truncated at the wide waveguide; (c) stable and (d) unstable surface solitons in the array truncated at the narrow waveguide.

Fig. 3. Families of discrete unstaggered surface light bullets in the binary array truncated at the wide waveguide (first curve from the left in both panels), and in the waveguide array truncated at the narrow waveguide (second curve from the left in both panels). (a) Power vs. propagation constant. Points (a)-(d) mark the modes shown in Figs. 2(a-d), respectively. (b) Hamiltonian vs. power.

correspond to the lower branch of the dependence $H = H(P)$. The typical single cusp-behavior of the dependence $H = H(P)$ is shown in Fig. 3(b) where the lower branches correspond to the stable surface modes. This observation is confirmed by direct simulations of the propagation of the stationary solitons perturbed by a white input noise. We also notice that there are no solutions for $\sigma \to 0$ and therefore no linear surface modes exist in the linear limit and the excitation of the nonlinear surface modes require a finite power for their excitation.

The stability results, which follow directly from the dependence $H = H(P)$ shown in Fig. 3(b), have been cross-checked in direct simulations of the dynamical equations (1) carried...
out by means of the Crank-Nicholson scheme, where transparent boundary conditions were implemented in order to permit the escape of radiation from the computation window. The system of nonlinear finite-difference equations is solved first by means of the Picard iteration method [19], and the resulting linear system is treated using the Gauss-Seidel iterative scheme. For a good convergence we need typically five Picard iterations and six Gauss-Seidel iterations. We have employed a transverse grid with the step-size \( \Delta \tau = 0.02 \), and used a typical longitudinal step-size of \( \Delta z = 2 \times 10^{-4} \).

We have revealed that in the case of unstaggered TIR solitons the eigenvalues are either real or pure imaginary. These results correspond well to the predictions made with the help of the \((H, P)\) diagram. By direct numerical simulations, we have found that the interface \((n = 0)\) unstable solitons reshape to stable solitons located on the same site, pertaining to the lower (stable) branch of the soliton family.

![Fig. 4] Fig. 4. (a,c) Stable (for \( \beta = -0.45 \) and \( \beta = 0.45 \), respectively) and (b,d) unstable (for \( \beta = -0.1 \) and \( \beta = 0.1 \), respectively) nonlinear spatiotemporal surface gap solitons corresponding to the arrays truncated at the narrow and wide waveguides, respectively.

4. Surface gap light bullets

The linear spectrum of the binary waveguide array consists of two different bands separated by a gap defined by the difference between two types of waveguides (see details in Refs. [14, 15]). The spatiotemporal discrete surface solitons studied above are located in the TIR gap, and they have unstaggered profiles. A novel type of spatial solitons, gap solitons, appears in the spectral gap due to the Bragg reflection [14, 15]. Therefore, spatiotemporal surface solitons in the BR gap can naturally be termed Surface gap light bullets; such solitons should have staggered profiles, and they can be associated with the surface Tamm states, similar to the case of waveguide arrays with defocusing nonlinearity [3, 4].

Figures 4(a-d) show several examples of the staggered spatiotemporal continuous-discrete localized states which describes the surface gap light bullets, with the propagation constant...
located in a gap of the linear spectrum. We notice that the stability domain of such solitons is rather narrow, and the examples of the stable solitons shown in Figs. 4(a,c) correspond to the propagation constants selected very close the edge of the existence domain.

![Power vs. propagation constant diagram](image)

Fig. 5. Power vs. propagation constant diagram for the surface gap light bullets (spatiotemporal nonlinear Tamm states) corresponding to (a) an array truncated at a narrow waveguide, and (b) an array truncated at a wide waveguide. Points (a-d) correspond to the soliton profiles shown in Figs. 4(a-d), respectively. (c,d) Real part of the dominant instability eigenvalue vs. propagation constant for the corresponding case (a,b) shown above.

The spatiotemporal surface gap solitons existing in the BABA... truncated waveguide structure become unstable for \( \beta \geq \beta_{cr} = -0.445 \) [see Figs. 5(a,c)], whereas the surface gap solitons in the ABAB... truncated structure becomes unstable for \( \beta \leq \beta_{cr} = 0.445 \), as follows from Figs. 5(b,d). For these staggered surface solitons the eigenvalues are complex and, therefore, the soliton dynamics is associated with the so-called oscillatory instabilities. The green dots in Figs. 5(c,d) separate stable and unstable branches. We have verified additionally that the stable solitons resisted to a 5% white noise, whereas the unstable solitons reshape to stable solitons.

5. Conclusions

We have analyzed the existence and stability of spatiotemporal surface solitons in one-dimensional binary arrays of weakly coupled nonlinear waveguides. We have combined the key features of both continuous (in time) and discrete (in space) nonlinear modes and analyzed, for the first time to our knowledge, the properties of continuous-discrete soliton families describing spatiotemporal discrete surface solitons in the binary waveguide arrays. We have revealed the existence of two different classes of such spatiotemporal surface solitons including a novel class of surface gap light bullets. Excitation of these spatiotemporal surface solitons by short pulses near the surface is a separate issue, and it requires further studies.
Acknowledgments

This work was supported by the Australian Research Council and the German Federal Ministry of Education and Research (FKZ 13N8340/1), the Deutsche Forschungsgemeinschaft (Priority Program SPP1113, FOR557 and Research Unit 532).