Photon-pair generation in arrays of cubic nonlinear waveguides

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Abstract: We study photon-pair generation in arrays of cubic nonlinear waveguides through spontaneous four-wave mixing. We analyze numerically the quantum statistics of photon pairs at the array output as a function of waveguide dispersion and pump beam power. We show flexible spatial quantum state control such as pump-power-controlled transition between bunching and anti-bunching correlations due to nonlinear self-focusing.

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References and links
1. Introduction

With the development of quantum optics it is now feasible to realize complex quantum logic algorithms, however an increasing number of optical elements is required for multi-step quantum simulations. Integrated photonic circuits have been considered as the solution to this challenge, as they are intrinsically scalable and interferometrically stable [1]. Integrated realization of multi-photon entanglement [1], quantum factoring algorithms [2], and polarization entanglement [3] have already been demonstrated experimentally.

One of particularly interesting devices in integrated photonics is a waveguide array (WGA). Recently WGs have been shown to generate unusual and strongly non-classical correlations of photon pairs propagating in the regime of quantum walks [4]. Combining quantum walks with photon pair generation in nonlinear waveguide arrays further opens the possibility for enhanced spatial quantum state control and improved clarity of spatial correlations [5]. While WGs with quadratic nonlinearity have been recently studied [5, 6], we expect that WGs with cubic nonlinearity can provide an entirely new realm of all-optical control of the quantum photon statistics. With on-chip photon-pair sources based on spontaneous four-wave mixing (SFWM) being readily available [7, 8], it becomes of much interest to study the quantum state dynamics in WGs with cubic nonlinear response.

In this work, we describe the generation of correlated photon pairs through SFWM in a cubic nonlinear WGA and analyze the interplay between SFWM phase-matching and WGA dispersion for the generation of complex entangled quantum states. We also demonstrate the potential of Kerr-based self-phase modulation (SPM) and cross-phase modulation (XPM) for quantum state control by investigating a case of stronger pump with special spectral filtering.

2. SFWM in WGs at low pump powers

We consider lossless near-degenerate SFWM with signal and idler frequencies being close to the pump frequency, with the rest of photon-pairs being filtered out. In this case the mode profiles and coupling between waveguides remain approximately the same for pump, signal and idler photons. It has been demonstrated that even with less than 0.5% difference between these frequencies, the pump can be effectively filtered at the output of the system, and photon pair correlations can be measured with high signal-to-noise ratio [7, 8]. The main difference between spontaneous four-wave mixing in bulk or single waveguides in comparison to WGs [Fig. 1(a)] is a different spatial dispersion leading to modified phase-matching. We begin the analysis of SFWM in WGA by studying the four-wave-mixing phase-matching for plane waves, \( \Delta \beta = 2\beta_p - \beta_s - \beta_i \). Here \( \beta_{p,s,i} \) are the propagation constants for pump, signal and idler. In a WGA they depend on normalized transverse momenta \( k_{p,s,i} \) as \( \beta_{p,s,i} = \beta^{(0)}_{p,s,i} + 2C\cos(\pi k_{p,s,i}) \), where \( C \) is the coupling coefficient between the waveguides [9]. Meanwhile the overlap between interacting Bloch waves [5, 9] can be written as \( \sum_n \exp[\pi (2k_p^+-k_s^+-k_i^+)n] \), where \( n \) is a waveguide number. Since \( \sum_n \exp(2\pi k_p^+n - i\pi k_s^+n - i\pi k_i^+n) = 2\pi \sum_N \delta(2\pi k_p^+ - \pi k_s^+ - \pi k_i^+ - 2\pi N) \), where \( N \in \mathbb{Z} \), then the transverse momenta \( k_{p,s,i}^+ \) for periodical solutions will satisfy the condition \( 2k_p^+ = k_s^+ + k_i^+ \). Therefore we can write an analytical expression for plane-wave four-wave mixing phase-mismatch in a WGA:

\[
\Delta \beta = \Delta \beta^{(0)} + 4C\cos(\pi (k_p^++k_i^+))/2 - 2C\cos(\pi k_i^+) - 2C\cos(\pi k_s^+).
\]

The spatial dispersion and therefore the phase-matching conditions for SFWM in WGs are qualitatively different from that in bulk, opening new possibilities for generating photon pairs with unusual quantum statistics.
We notice that the spatial dispersion described by Eqs. (2) and (3) exactly agrees with Eq. (1).

Generation of photon pairs in cubic nonlinear WGAs through SFWM in the absence of multiple waveguides, it then propagates in the regime of discrete diffraction [9] as illustrated in Fig. 1(b). For developing a model describing SFWM based photon-pair generation with a negligible number of higher multiphoton events in a cubic WGA, we first consider low pump powers that do not lead to pump spatial reshaping or nonlinear phase modulation. In the absence of SPM, XPM and losses, the system Hamiltonian consists of linear [4] and nonlinear [10] parts \( H = H^{(\text{lin})} + H^{(\text{nonlin})} \), \( H^{(\text{lin})} = i\hbar \sum_{n} \Delta \beta^{(0)} \hat{a}_{n}^{\dagger} \hat{a}_{n} + \text{C}\hat{a}_{n-1}^{\dagger} \hat{a}_{n} + \text{C}\hat{a}_{n+1}^{\dagger} \hat{a}_{n} \) and \( H^{(\text{nonlin})} = \gamma \hbar \sum_{n} \left[ E_{n_{p}}^{(p)}(z)E_{n_{p}}^{(p)*} \hat{a}_{n_{p}}^{\dagger} \hat{a}_{n_{p}} - E_{n_{p}+1}^{(p)*}E_{n_{p}}^{(p)} \hat{a}_{n_{p}}^{\dagger} \hat{a}_{n_{p}} - E_{n_{p}}^{(p)*}E_{n_{p}-1}^{(p)} \hat{a}_{n_{p}}^{\dagger} \hat{a}_{n_{p}} \right] \). Here \( n_{s} \) and \( n_{i} \) are the waveguide numbers describing the positions of the signal, and the idler photons, and \( E_{n_{p}}^{(p)}(z) \) is the pump amplitude in waveguide number \( n_{p} \). \( \Delta \beta^{(0)} \) is the linear four-wave mixing phase-mismatch in a single waveguide, \( \gamma \) is a nonlinear coefficient. The normalized pump field profile evolution along the propagation distance \( z \) is defined through the classical coupled-mode equations [9]:

\[
dE_{n_{p}}^{(p)}(z)/dz = iC \left[ E_{n_{p}-1}^{(p)}(z) + E_{n_{p}+1}^{(p)}(z) \right].
\]  

Generation of photon pairs in cubic nonlinear WGAs through SFWM in the absence of multiple photon pairs can be characterized by the evolution of a bi-photon wave function \( \psi_{n_{s},n_{i}}(z) \) in a Schrödinger-type equation. The equation is obtained from the Hamiltonian, and it has a form similar to that of quadratic media [11]:

\[
d\psi_{n_{s},n_{i}}(z)/dz = iC \left[ \psi_{n_{s}-1,n_{i}}(z) + \psi_{n_{s},n_{i}-1}(z) + \psi_{n_{s},n_{i}+1}(z) + \psi_{n_{s},n_{i}+1}(z) \right] + i \Delta \beta^{(0)} \psi_{n_{s},n_{i}} + \gamma E_{n_{p}}^{(p)}(z)E_{n_{p}}^{(p)*}(z) \delta_{n_{s},n_{i}}.
\]  

We notice that the spatial dispersion described by Eqs. (2) and (3) exactly agrees with Eq. (1).

After calculating the wave function, we obtain two-photon correlations in real-space as \( \Gamma_{n_{s},n_{i}} = |\psi_{n_{s},n_{i}}(L)|^2 \), where \( L \) is the propagation length. In order to find correlations for the signal and idler photons in \( k \)-space we apply the two-dimensional Fourier-transform, \( \Gamma_{k_{x},k_{y}} = \sum_{n_{s}} \sum_{n_{i}} \exp(i\pi k_{x} \cdot n_{s}) \exp(i\pi k_{y} \cdot n_{i}) \psi_{n_{s},n_{i}}(L)^2 \). For the examples presented below, we normalize

Fig. 1. (a) Schematic and (b) power profile of linear pump propagation in a WGA for input pump amplitude \( E_{n_{p}}^{(p)}(0) = 10^{-5} \). (c-f) Photon-pair correlations in (c,e,g) real-space and (d,f,h) \( k \)-space for different group-velocity dispersion: (c,d) anomalous \( \Delta \beta^{(0)} = -18 \), (e,f) zero \( \Delta \beta^{(0)} = 0 \) and (g,h) normal \( \Delta \beta^{(0)} = 18 \).
all parameters to the WGA length $L = 1$ and nonlinearity $\gamma = 1$. The physical value of the nonlinear coefficient can be determined following the approach of Ref. [10]. We use the coupling coefficient $C = 5$, and consider a pump beam coupled only to the central waveguide ($n = 0$).

We study the case of low pump amplitude $E_{np=0}(0) = 10^{-5}$ when the input beam exhibits linear discrete diffraction [9], see Fig. 1(b). We analyze three different types of group velocity dispersion (GVD): anomalous $\Delta \beta^{(0)} = -18$, zero $\Delta \beta^{(0)} = 0$ and normal $\Delta \beta^{(0)} = 18$. In the case of anomalous GVD, photons in a pair tend to end up mostly away from the central waveguide, with higher probability to be at either the same or the opposite waveguides, see photon-pair probability correlation in Fig. 1(c). This behavior corresponds to weakly pronounced simultaneous spatial bunching and antibunching. The quantum statistics in this case is quasi-anyonic, which can be interesting for quantum simulations [12]. The k-space correlations show an elliptical shape centered at $k^+_{s} = k^+_{i} = 0$ [Fig. 1(d)]. This shape corresponds to the wavenumbers with the most efficient phase-matched interactions, i.e. $\Delta \beta = 0$. For zero GVD the signal and idler photons mostly leave the structure from the same waveguides, thus demonstrating strong spatial bunching behavior [Fig. 1(e)]. Figure 1(f) shows that the transverse wavenumbers for photon pairs satisfy the relations $k^+_{s} + k^+_{i} \simeq \pm 1$. We note that for zero dispersion photon pairs have much higher probability to arrive to the center of the WGA, because phase matching can now be achieved for a broader range of transverse momenta. In the case of normal GVD, the real-space correlations [Fig. 1(g)] are the same as for anomalous GVD [Fig. 1(c)]. Indeed for low pump powers with negligible SPM and XPM, the system is symmetrical with respect to the GVD sign. In k-space the correlations form an elliptical shape centered around $k^+_{s} = k^+_{i} = \pm 1$ [Fig. 1(h)]. We note that there is a gradual transition from ellipses centered at 0 [Fig. 1(d)] through linear shapes [Fig. 1(f)] to ellipses centered at $k^+_{s} = k^+_{i} = \pm 1$ [Fig. 1(h)] when tuning the GVD from anomalous to normal. GVD tuning can achieved by changing the pump wavelength [13], however such tuning can be complicated due to the required corresponding spectral shift of output filters for signal and idler photons.

3. SFWM and pump self-focusing at high pump powers

Next we investigate the potential of SPM and XPM at high pump powers for flexible quantum state control. When the pump power is increased, the beam self-focusing results in a sharp transition from discrete diffraction to the formation of a spatial soliton, as shown in Fig. 2(a). However, the powers required for soliton formation are at least an order of magnitude higher than those needed to remain in the regime with small number of multiphoton events. For example, in a 3 mm long Si WGA (waveguides 200 – 300 nm high and 400 – 500 nm wide) the pump peak power required for noticeable nonlinear phase modulation is of the order of $10 – 20$ W [14]. The characteristic pump peak power $P$ for photon-pair generation is much smaller: in a 10 mm long Si waveguide $P \approx 0.1 – 0.2$ W [7, 8] (corresponding to $1 – 2$ W for a 3 mm long waveguide).

To realize the influence of SPM and XPM on the photon-pair generation in WGAs in the ab-
Fig. 3. (a) Pump spatial soliton formation with input pump amplitude $E_{np=0}^{(p)}(0) = 4.5$.
(b-g) Photon-pair correlations in (b,d,f) real space and (c,e,g) k-space for different group-velocity dispersion: (b,c) anomalous $\Delta \beta^{(0)} = -18$, (d,e) zero $\Delta \beta^{(0)} = 0$ and (f,g) normal $\Delta \beta^{(0)} = 18$.

To avoid the presence of multiphoton events, we suggest asymmetric filtering approach for a pulsed pump beam of transform-limited pulses having a spectrum shown in Fig. 2(b) with a green solid line. In this approach we choose two narrowband spectral filters for measuring the signal (red dashed line) and the idler (blue dotted line) photons, such that they are located asymmetrically with respect to the central pump frequency. Then only a narrow window of pump frequencies with small peak power (indicated by gray shading) would be responsible for the detected photon-pairs, due to the energy conservation $\omega_p + \omega_i = \omega_s + \omega_i^{\text{filtered}}$. With such filtering the pump peak power can be strong enough to induce pump beam self-focusing, while multiphoton events are mostly excluded from the measurement. The complete modeling of this system should account for the spatio-temporal pump dynamics. Here we investigate a simplified steady-state model that is valid if the pump pulse does not experience any dispersion-related reshaping. Signal and idler filters in this case should be far enough from the pump in frequency domain, so that the phase-matching is affected by XPM and SPM, but also close enough, so that the coupling coefficients are still similar for signal, idler and pump waves. Since coupling dispersion in WGAs is usually smaller than the temporal dispersion (unless specifically engineered otherwise [15]), these assumptions should be valid for a large variety of systems.

Since high pump peak powers lead to the pump beam focusing [Fig. 2(a)], generated photon pairs will have different spatial distributions depending on the pump power. The pump power will also effectively change the SFWM phase-matching conditions due to XPM. We incorporate these effects by adding the terms responsible for XPM and SPM into the Eqs. (2) and (3):

$$
\frac{dE_{np}^{(p)}(z)}{dz} = iC \left[ E_{np-1}^{(p)}(z) + E_{np+1}^{(p)}(z) \right] + i\gamma |E_{np}^{(p)}(z)|^2 E_{np}^{(p)}(z),
$$

$$
\frac{d \psi_{n_s,n_i}(z)}{dz} = iC \left[ \psi_{n_s-1,n_i}(z) + \psi_{n_s,n_i-1}(z) + \psi_{n_s+1,n_i}(z) + \psi_{n_s,n_i+1}(z) \right] + \gamma |E_{n_s}^{(p)}(z)|^2 \delta_{n_s,n_i} + 1 \left[ \Delta \beta^{(0)} + 2\gamma |E_{n_s}^{(p)}(z)|^2 + 2\gamma |E_{n_i}^{(p)}(z)|^2 \right] \psi_{n_s,n_i}(z),
$$

We acknowledge that this model is only the first-order approximation and that simulations designed to give precise quantitative results should incorporate full dispersion curves both for propagation and coupling constants, as well as spatio-temporal dynamics. However, we believe that the simplified model that we present here is useful to obtain a qualitative insight into the quantum statistics control that can be achieved in cubic WGAs.
Fig. 4. (Media 1) (a) Photon-pair bunching to antibunching ratio $R$ vs. the input pump amplitude $E_{n_p=0}^{(p)}(0)$ for the anomalous GVD $\Delta \beta^{(0)} = -18$ (red solid line) and the normal GVD $\Delta \beta^{(0)} = 18$ (blue dashed line). Red and blue circles correspond to the input pump amplitude $E_{n_p=0}^{(p)}(0) = 2.850$ for the plots (b-d). (b,c) Real-space photon-pair correlations at (b) anomalous and (c) normal dispersion. (d) Pump power distribution in the array.

When the pump power is increased, the pump beam distribution collapses to a single waveguide, as illustrated in Fig. 3(a) for $E_{n_p=0}^{(p)}(0) = 4.5 [E_{n_p=0}^{(p)}(0)/C = 0.9]$. This self-focusing dramatically changes the spatial photon-pair correlations. The real space correlations in anomalous GVD regime collapse to a single waveguide, following the pump [Fig. 3(b)]. This can be explained by the phenomenon of forward-propagation dispersion compensation. Anomalous GVD allows to phase-match four-wave mixing in a single waveguide, which corresponds to a broad k-space phase matching in the array [Fig. 3(c)]. For zero GVD, the XPM leads to a phase-matching of angled SFWM, corresponding to a small-scale antibunching in real space correlations [Fig. 3(d)] and more pronounced selectivity of phase-matched signal and idler transverse momenta [Fig. 3(e)]. In the normal GVD case the real space correlations now demonstrate very pronounced antibunching [Fig. 3(f)], which corresponds to k-space phase matching mostly for transverse momenta far away from zero [Fig. 3(g)]. This makes it possible for photon pairs to escape from the localized pump beam. In this case the generated photon pairs become spatially filtered from the pump beam and show strongly pronounced anti-bunching. This demonstrates that by tuning the pump wavelength and power we can get a great degree of control on the spatial photon-pair correlations, which is useful for applications in quantum information.

To further illustrate this flexibility, we focus on two particularly interesting regimes, namely bunching and antibunching. We introduce a bunching to antibunching ratio, $R = (\sum_{n_\gamma=n_\gamma} \Gamma_{n_\gamma,n_\gamma})/(\sum_{n_\gamma=-n_\gamma} \Gamma_{n_\gamma,n_\gamma})$ and study the dynamics of this ratio with respect to pump power [Fig. 4(a) (Media 1)], while also tracking the spatial photon-pair correlations for anomalous [Fig. 4(b) (Media 1)] and normal GVD [Fig. 4(c) (Media 1)] and the pump propagation [Fig. 4(d) (Media 1)]. We observe that the pump power tuning provides access to a wide range of photon-pair quantum statistics. We also see that in general, normal dispersion can lead to stronger spatial antibunching, as in this case only angled phase-matching can be satisfied.

4. Conclusion
We have demonstrated that waveguide arrays with cubic nonlinearity can be employed as a flexible platform for all-optical manipulation of the generated bi-photon quantum statistics. We have shown that interplay between spatial dispersion and four-wave mixing phase-matching in waveguide arrays leads to the generation of complex spatial quantum states, which can be controlled by tuning the pump power and wavelength. We anticipate that our results will open new opportunities for integrated quantum photonics with all-optical controls.

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