Multivortex solitons in triangular photonic lattices

Tristram J. Alexander, Anton S. Desyatnikov, and Yuri S. Kivshar

Nonlinear Physics Centre and Centre for Ultra-high Bandwidth Devices for Optical Systems (CUDOS), Research School of Physical Sciences and Engineering, Australian National University, Canberra ACT 0200, Australia

Received January 2, 2007; accepted February 12, 2007;
posted February 27, 2007 (Doc. ID 78584); published April 17, 2007

We introduce a novel class of stable lattice solitons with a complex phase structure composed of many single-charge discrete vortices in a triangular photonic lattice. We demonstrate that such nonlinear self-trapped states are linked to the resonant Bloch modes, which bear a honeycomb pattern of phase dislocations.

OCIS codes: 190.4420, 190.5530.

Among various transverse effects in nonlinear optics, the light control in periodic photonic structures is receiving a growing interest.[1] Dramatic changes of the diffraction and self-focusing of light in media with periodically modulated refractive index offer novel directions for routing and switching in all-optical photonic devices. One of the interesting problems is to shape the transverse structure of light beams in the lattices and generate self-trapped phase singularities and optical vortices[2] in the form of the discrete vortex solitons[3–6] first generated experimentally in square photonic lattices.[7,8]

Lattice (or discrete) solitons appear as spatially localized states of light in periodic photonic lattices and can be associated with self-trapping of Bloch waves and existing in the gaps of the linear bandgap spectrum. The structure and symmetry of the associated Bloch waves then determine the properties of the corresponding lattice solitons. For example, the fundamental discrete soliton corresponds to the amplitude-modulated Bloch wave in the total-internal-reflection semi-infinite gap, and it originates from the Γ-point of the linear spectrum. The discrete vortex solitons in a square lattice can be presented as a superposition of two $\pi/2$-out-of-phase Bloch waves, similar to self-trapped vortex lattices.[9] Nonlinear Bloch waves share the bands of the spectrum with linear Bloch waves, and they became localized in the gaps in the form of truncated nonlinear Bloch waves[10,11] (TNBWs). Discrete solitons can be viewed as the elementary structures at the limit of such truncation.

In this Letter we analyze the existence of higher-order states with multivortex phase structure. We show that, in a sharp contrast to the square lattices studied earlier, triangular photonic lattices with the threefold symmetry can support multivortex discrete localized states. Such localized modes originate from the specific type of (linear and nonlinear) Bloch waves that carry a periodic pattern of phase dislocations. Surprisingly, we find stable extended localized states with higher vorticity (discrete vortex clusters), whereas their counterparts with lower topological charge experience topological instabilities.[12] We predict the existence of a variety of singular discrete solitons by solving the power balance equations and then find all such multivortex states by a numerical relaxation technique,[13] verifying their stability in direct numerical simulations.

We consider the propagation of optical beams in a medium with a periodically modulated refractive index and photorefractive-type saturable nonlinearity described by the paraxial nonlinear Schrödinger equation[1] for the slowly varying envelope Ψ of the electric field:

$$i\frac{\partial\Psi}{\partial z}+D\left(\frac{\partial^2\Psi}{\partial x^2}+\frac{\partial^2\Psi}{\partial y^2}\right)-\frac{I_0\Psi}{1+V(x,y)+|\Psi|^2}=0, \quad (1)$$

where the propagation variable z and transverse coordinates (x,y) are measured in millimeters and micrometers, respectively, so that the diffraction coefficient $D=18.015$ and the coefficient of nonlinearity $I_0=2.36$.[14] The lattice is optically induced[15,16] by the linear periodic diffraction-free wave with the intensity distribution given by

$$V(x,y)=2V_0\left[\frac{3}{2}+\cos(\mathbf{k}_{12}\mathbf{r})+\cos(\mathbf{k}_{13}\mathbf{r})+\cos(\mathbf{k}_{23}\mathbf{r})\right],$$

where $\mathbf{k}_{12}=k_0\{1,1/\sqrt{3}\}$, $\mathbf{k}_{13}=k_0\{1,-1/\sqrt{3}\}$, and $\mathbf{k}_{23}=k_0\{0,2/\sqrt{3}\}$ with $k_0=2\pi/d$, d being the lattice spacing. In experiment, such hexagonal patterns can be generated[9,17] by interfering three plane waves with intensity V_0 and transverse wave vectors $\mathbf{k}_i$, forming a triangle [red dots in Fig. 1(a)] so that $\mathbf{k}_{ij}=\mathbf{k}_i-\mathbf{k}_j$.

We look for the stationary solutions of Eq. (1) in the form $\Psi(x,y,z)=\psi(x,y)\exp(i\beta z)$, where β is the nonlinearity-induced shift of the propagation constant. In the linear limit, the stationary solutions are the (linear) Bloch waves, $\psi(\mathbf{r})=\phi_{\mathbf{k}}(\mathbf{r})\exp(i\mathbf{k}\mathbf{r})$, where the wave vector $\mathbf{k}$ is selected in the first Brillouin zone [see Fig. 1(a)] and $\phi_{\mathbf{k}}(\mathbf{r})$ is a periodic function. The spectrum $\beta(\mathbf{k})$ of the linear Bloch waves has the band structure shown in Fig. 1(c). At the center of the Brillouin zone (Γ-point) the Bloch wave has trivial phase and amplitude modulated by the lattice [see Fig. 1(b)]. However, the Bloch wave at the band edge [M-point in Figs. 1(a) and 1(c)] has a phase with the structure of a vortex lattice [see Fig. 1(e)]. This nontrivial Bloch wave can be excited through the resonant Zener tunneling in the tilted lattice.[18]

The fundamental localized stationary solutions are the bright solitons[17] positioned on the sites of the lattice [Fig. 1(f)]. Such self-trapped states exist because of self-focusing nonlinearity; however, their particu-

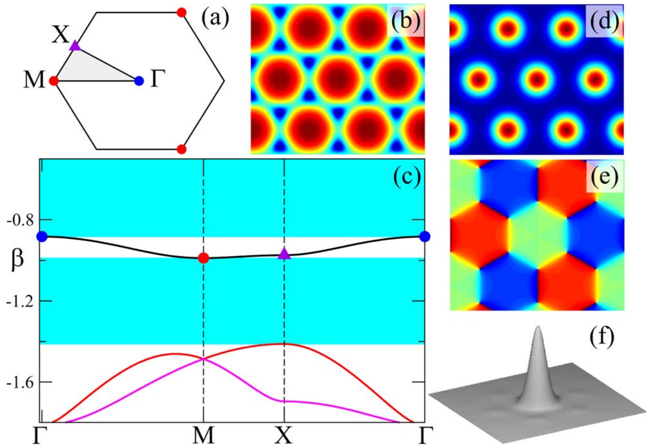

Fig. 1. (Color online) (a) First Brillouin zone, (b) intensity, and (c) bandgap spectrum for a triangular lattice (gaps are shaded) for $V_0=0.6$. (d) Intensity and (e) phase of the Bloch wave at the M-point. (f) Fundamental soliton originating from the Γ-point.[17]

lar structure depends on the lattice symmetry. Complex stationary configurations on the lattice can be constructed as a superposition of the fundamental solitons with specific phase structure.[19,20] The latter will determine the exchange of energy between individual sites, and thus the energy flow within the cluster.[21] This formalism results in a set of coupled equations for the phase of the n-th soliton ϕ_n ($n=1,\ldots N$):

$$\sum_{j=1}^{N} c_{nj}\sin(\phi_n-\phi_j)=0, \qquad (2)$$

where c_{nj} are the coupling coefficients for two solitons determined by nonlinearity as well as symmetry of the lattice.[20] We solve Eq. (2) for different soliton configurations and find the structure of the allowed states and possible phase relations. The simplest singular state is a three-site vortex shown in Fig. 2(b). Because of the symmetry, the coefficients in Eq. (2) coincide, $c_{nj}\equiv c$; thus the phases $(0,2\pi/3,4\pi/3)$ of the coupled solitons are determined solely by the lattice geometry, and they are the same as in the case of a stationary azimuthon.[22]

The analysis described above predicts the stationary states that we then find numerically by minimizing the error functional.[13] A more general approach[10,11] allows exploring the relations between linear and nonlinear waves guided by the lattice. Namely, the nonlinearity-induced extension of every linear Bloch wave is an infinite periodic nonlinear Bloch wave, with the amplitude varying with β. The dashed curves in the bottom panel of Fig. 2 show two examples of the nonlinear Bloch waves in the total-internal-reflection gap; the one with the larger amplitude is singular, and we refer to it as the M-Bloch wave, while the one with smaller amplitude, the Γ-Bloch wave, has a trivial phase.

The nonlinear Bloch wave can be made localized either in the band gaps[10] or, for the case of attractive nonlinearity studied here, in the semi-infinite gap.[11] Figures 2(a)–2(c) present three examples of such localized states at the same values of $\beta=-0.8$, $d=30$, and $V_0=0.6$. First, in Fig. 2(a) we show the truncation of the ground-state Γ-Bloch wave with an imposed vortex phase; we refer to it as Γ-TNBW because it has an amplitude close to the nonlinear Γ-Bloch wave and its Fourier image is located close to the origin of Brillouin zone (Γ-point). The amplitude of the fundamental vortex in Fig. 2(b) is between two nonlinear Bloch waves and so is its Fourier image; although it is much broader because of stronger localization, the maxima of the three spots are located between the origin and the edge of the Brillouin zone. Finally, the novel multivortex soliton in Fig. 2(c) has an amplitude approaching the nonlinear M-Bloch wave and its Fourier image occupies three M-points, similar to the lattice wave. Thus the number of vortices within TNBW determines its position in the Brillouin zone and, as we demonstrate below, defines its stability.

The stability properties of the novel multivortex solitons are somewhat surprising. In Fig. 3 we compare the dynamics of two TNBW states occupying the same lattice sites but carrying different vorticity. One would expect[2] that the soliton with higher total topological charge in Figs. 3(a) and 3(b) will be less stable than its lower-order counterpart in Figs. 3(c) and 3(d). In a sharp contrast, we find that the vortex clusters are generally stable, regardless of the number of vortices they carry, while TNBWs with a single vor-

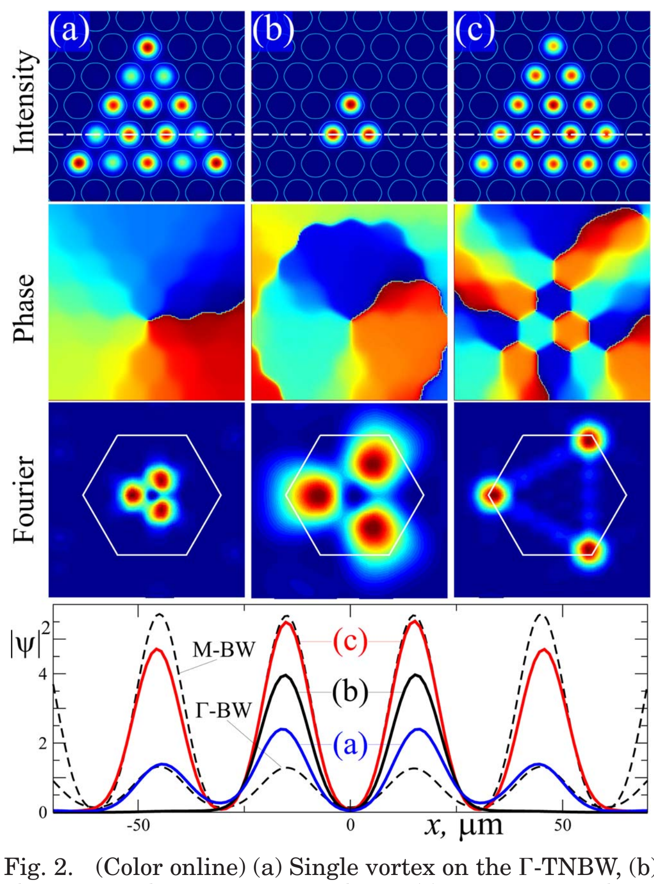

Fig. 2. (Color online) (a) Single vortex on the Γ-TNBW, (b) elementary discrete vortex soliton, (c) M-TNBW with 24 vortices and total charge +4. Bottom panel shows the intensity profiles through the horizontal cut compared with nonlinear Bloch waves (dashed curves).

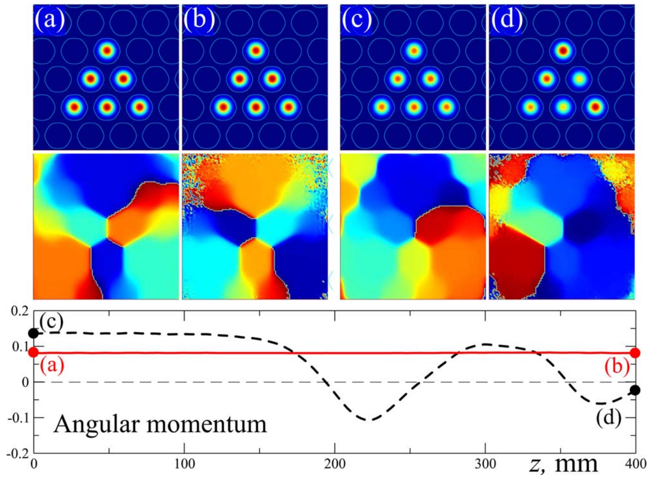

Fig. 3. (Color online) Dynamics of higher-order vortex solitons and evolution of their angular momenta: (a), (b) composed of four vortices with the total charge $m=+2$ (solid line) and (c), (d) single vortex with $m=+1$ (dashed line).

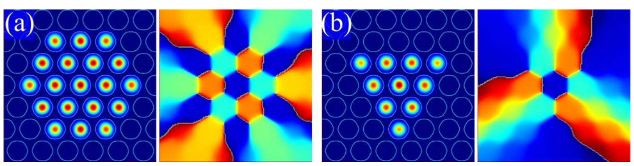

Fig. 4. (Color online) Two examples of M-TNBW vortex solitons. (a) Hexagonal, 24 vortices, zero charge. (b) Inverted triangle, 9 vortices, total charge $m=-3$.

tex are unstable. This is a consequence of the modulational instability of the corresponding nonlinear Γ-Bloch wave. Even when the soliton is strongly trapped by the lattice, as in the case shown in Figs. 3(c) and 3(d), there appears a phase instability that destroys its topological structure, similar to the charge-flipping instability[20] and topological transformations[12] observed in square lattices. Increasing the size of the truncated state leads to more pronounced instability and intensity changes.

In the case of vortex clusters, the symmetry and size of the truncation determine the total charge of the state. As can be seen in Fig. 4(a), the TNBW with a hexagonal symmetry always has zero total charge, despite the many vortices it contains. In the triangular TNBW, as in Fig. 4(b), the total charge is given by $(N-1)$, where N is the number of lobes on the side of the triangle; the sign of the total charge is determined by the orientation of the triangle. For instance, inverting the triangle in Fig. 4(b) leads to a state with the charge −3. Both types of the vortex clusters shown in Fig. 4 can be stable.

In conclusion, we have introduced and studied novel classes of extended self-trapped states of light in triangular photonic lattices in the form of multivortex lattice solitons. We have demonstrated that such nonlinear localized states are linked to the Bloch modes, which are characterized by a honeycomb pattern of phase dislocations.

The work has been supported by the Australian Research Council and Australian National University. The authors thank Andrey A. Sukhorukov for useful discussions and suggestions. A. S. Desyatnikov's e-mail address is asd124@rsphysse.anu.edu.au.

References

1. Yu. S. Kivshar and G. P. Agrawal, *Optical Solitons* (Academic, 2003).
2. A. S. Desyatnikov, Yu. S. Kivshar, and L. Torner, Prog. Opt. **47**, 291 (2005).
3. P. G. Kevrekidis, B. A. Malomed, and Yu. B. Gaididei, Phys. Rev. E **66**, 016609 (2002).
4. J. Yang and Z. H. Musslimani, Opt. Lett. **28**, 2094 (2003).
5. B. B. Baizakov, B. A. Malomed, and M. Salerno, Europhys. Lett. **63**, 642 (2003).
6. J. Yang, New J. Phys. **6**, 47 (2004).
7. D. N. Neshev, T. J. Alexander, E. A. Ostrovskaya, Yu. S. Kivshar, H. Martin, I. Makasyuk, and Z. Chen, Phys. Rev. Lett. **92**, 123903 (2004).
8. J. W. Fleischer, G. Bartal, O. Cohen, O. Manela, M. Segev, J. Hudock, and D. N. Christodoulides, Phys. Rev. Lett. **92**, 123904 (2004).
9. A. S. Desyatnikov, N. Sagemerten, R. Fischer, B. Terhalle, D. Traeger, D. N. Neshev, A. Dreischuh, C. Denz, W. Krolikowski, and Yu. S. Kivshar, Opt. Express **14**, 2851 (2006).
10. T. J. Alexander, E. A. Ostrovskaya, and Yu. S. Kivshar, Phys. Rev. Lett. **96**, 040401 (2006).
11. T. J. Alexander and Yu. S. Kivshar, Appl. Phys. B **82**, 203 (2006).
12. A. Bezryadina, D. Neshev, A. S. Desyatnikov, J. Young, Z. Chen, and Yu. S. Kivshar, Opt. Express **14**, 8317 (2006).
13. J. J. García-Ripoll and V. Peŕez-García, SIAM J. Sci. Comput. **23**, 1316 (2001).
14. R. Fischer, D. Träger, D. N. Neshev, A. A. Sukhorukov, W. Krolikowski, C. Denz, and Yu. S. Kivshar, Phys. Rev. Lett. **96**, 023905 (2006).
15. N. K. Efremidis, S. Sears, D. N. Christodoulides, J. W. Fleischer, and M. Segev, Phys. Rev. E **66**, 046602 (2002).
16. D. Neshev, E. Ostrovskaya, W. Krolikowski, and Yu. S. Kivshar, Opt. Lett. **28**, 210 (2003).
17. C. R. Rosberg, D. N. Neshev, A. A. Sukhorukov, W. Krolikowski, and Yu. S. Kivshar, Opt. Lett. **32**, 397 (2007).
18. A. S. Desyatnikov, Yu. S. Kivshar, V. S. Shchesnovich, S. B. Cavalcanti, and J. M. Hickmann, Opt. Lett. **32**, 325 (2007).
19. Ya. V. Kartashov, A. A. Egorov, L. Torner, and D. N. Christodoulides, Opt. Lett. **29**, 1918 (2004).
20. T. J. Alexander, A. A. Sukhorukov, and Yu. S. Kivshar, Phys. Rev. Lett. **93**, 063901 (2004).
21. A. S. Desyatnikov and Yu. S. Kivshar, Phys. Rev. Lett. **88**, 053901 (2002).
22. A. S. Desyatnikov, A. A. Sukhoroukov, and Yu. S. Kivshar, Phys. Rev. Lett. **95**, 203904 (2005).