Dark-soliton stripes on a paraboloidal background in a bulk nonlinear medium

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We demonstrate self-similar evolution of fundamental and higher-order dark soliton stripes on a paraboloidal background in a bulk graded-index waveguide with self-defocusing nonlinearity. The results show that the dark soliton stripes can self-similarly evolve on the paraboloidal background when they pass through the nonlinear well or barrier, and their evolutions may lead to the generation of the optical vortex when the initial dark soliton stripes deviate from the central position of the paraboloidal background. In addition, the interactions between neighboring dark soliton stripes on the paraboloidal background are investigated, which exhibits some complicated vortex patterns.

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I. INTRODUCTION

Self-similarity of waves is a fundamental property that has been explored in many branches of physics, such as nonlinear optical systems, plasma physics, fluid dynamics, Bose-Einstein condensation, and condensed matter physics [1]. In general, self-similarity of a nonlinear system indicates the presence of an internal order, which implies that the wave profile remains unchanged and its amplitude and width simply vary with time or propagation distance. In nonlinear optics, the exact and the asymptotic self-similar solutions (also called optical similaritons), such as the exact optical similaritons in nonautonomous systems and the asymptotic parabolic optical similaritons in the gain amplifier systems, have been studied extensively due to their potential applications in nonlinearity and dispersion management systems. The exact optical similaritons are mainly described by the exact solitary wave solutions, including the bright and dark soliton solutions, the quasisoliton solutions, the solitary nonlinear Bloch waves, and the solitons on the continuous-wave background [2–18], whose existence requires delicate balances between the system parameters such as dispersion, nonlinearity, gain, and inhomogeneity. On the contrary, the asymptotic optical similaritons exist in a wide range of system parameters and are mainly described by the compact parabolic, Hermite-Gaussian, and hybrid functions, in which the parabolic similariton is more intriguing because they can retain the robustness for the higher initial power, and their linear chirp facilitates the efficient compression of optical waves [19–26].

Note that among all of these mentioned above, the study on dark solitons is less investigated. However, the experimentally successful demonstrations for dark solitons in various materials stimulate us to investigate the self-similar dynamics of dark solitons [27–30]. Recently, based on a stable stationary parabolic background existing in a planar graded-index waveguide with self-defocusing nonlinearity, we studied the self-similar evolution of dark solitons on a parabolic background in terms of \((1+1)\)-dimensional nonlinear Schrödinger (NLS) equation, and the results have shown that the dark solitons on the parabolic background can propagate stably and feature the properties similar to dark solitons for the standard NLS equation [31]. Compared with the evolutions of dark solitons on the Gaussian-type background [32–34], in which the Gaussian background spreads and the amplitudes of both dark beam and Gaussian background decrease during the propagation of dark solitons, our results may provide an efficient way to realize the propagation and the control of dark soliton in a waveguide with parabolic refractive index. For \((2+1)\)-dimensional homogeneous nonlinear media, it should be pointed out that dark soliton stripes have been studied [35–37], and it is shown that dark soliton stripes suffer from the transverse instabilities and eventually break into optical vortex solitons, which are stable (see the review papers [38,39], and references therein).

In this paper, we study the properties of dark soliton stripes in a bulk nonlinear medium with graded-index change. We first present an approximate self-similar solution with paraboloid type for \((2+1)\)-dimensional NLS equation by means of Thomas-Fermi approximation and demonstrate the self-similar evolution of fundamental and higher-order dark soliton stripes on the intense paraboloidal background in a bulk graded-index waveguide with self-defocusing nonlinearity. It is shown that the dark soliton stripes on the paraboloidal background can propagate stably and can self-similarly evolve on the paraboloidal background even when they pass through the nonlinear well or barrier. In addition, the interaction between neighboring dark soliton stripes on the paraboloidal background is investigated.

The paper is organized as follows. In Sec. II, the model describing beam propagation in a bulk graded-index nonlinear medium is presented and reduced into the standard \((2+1)\)-dimensional NLS equation with the external potential by the suitable transformations of variables and function. In Sec. III, an approximate self-similar solution with paraboloid type is presented by means of Thomas-Fermi approximation, and its properties will be discussed. In Sec. IV, we shall address the dynamics of fundamental and higher-order dark soliton stripes on the paraboloidal background in details. Section V discusses the interaction between neighboring...
dark soliton stripes on the paraboloidal background. Finally, Sec. VI presents conclusions.

II. MODEL AND REDUCTIONS

We start our analysis by considering the propagation of optical beams inside a bulk graded-index nonlinear waveguide amplifier with the refractive index

\[ n(z, x, y) = n_0 + n_1 F(z) r^2 + n_2 R(z) I(z, x, y), \]

where \( r^2 = x^2 + y^2 \) and \( I(z, x, y) \) is the optical intensity. Here the first two terms describe the linear part of the refractive index and the last term represents Kerr-type nonlinearity. For convenience, we assume that the second term \( n_1 > 0 \), and the function \( F(z) \) can be negative or positive, which corresponds to the graded-index waveguide acting as a focusing or defocusing lens. The Kerr parameter \( n_2 \) can take positive (negative) for nonlinear self-focusing (self-defocusing) medium, and the dimensionless function \( R(z) > 0 \), which represents inhomogeneity of Kerr nonlinearity along the medium. Under slowly varying envelope approximations, the nonlinear wave equation governing beam propagation in such an inhomogeneous waveguide given by Eq. (1) can be written as

\[ \frac{\partial u}{\partial z} + \frac{1}{2k_0} \nabla_{x,y}^2 u + \frac{k_0 n_1}{n_0} F(z) r^2 u + \frac{k_0 n_2}{n_0} R(z) |u|^2 u = \frac{i g(z)}{2} u, \]

where \( k_0 = 2\pi n_0 / \lambda \) is the wave number with \( \lambda \) being the wavelength of the optical source, \( \nabla_{x,y}^2 = \partial^2 / \partial x^2 + \partial^2 / \partial y^2 \) is the Laplacian, and \( g(z) \) is the gain (loss) coefficient \([g(z) > 0 \text{ for gain and } g(z) < 0 \text{ for loss}]).\) Introducing the normalized transformations \( X = x / w_0, \ Y = y / w_0, \ Z = z / L_D, \) and \( U = \sqrt{k_0} n_2 |L_D| / n_0 \partial u \), \( G(Z) = g(z) L_D \), where \( w_0 = (2 k_0 n_1 / n_0)^{-1/4} \) and \( L_D = k_0 w_0^2 \) represent the characteristic transverse scale and the diffraction length, respectively. Thus Eq. (2) can be rewritten in a dimensionless form

\[ \frac{\partial U}{\partial \eta} + \frac{1}{2} \nabla_{x,y}^2 U + \frac{1}{2} F(X^2 + Y^2) U + \sigma R(U)|U|^2 U = \frac{i G}{2} U, \]

where \( \sigma = n_1 / |n_2| = \pm 1 \) corresponds to self-focusing (+) and self-defocusing (−) nonlinearity of the waveguide, respectively; \( \nabla_{x,y}^2 = \partial^2 / \partial x^2 + \partial^2 / \partial y^2 \), and \( F(Z), R(Z), \) and \( G(Z) \) are the functions of the normalized distance \( Z \). It is instructive to consider a suitable experimental apparatus, in which the graded refractive index \( n_1 \) (for example, \( n_1 = 0.1 \text{ cm}^{-2} \) [7]) in Eq. (1) leads to the characteristic transverse scale \( \omega_0 \approx 34 \mu m \) and the diffraction length \( L_D \approx 3.7 \text{ cm near } 532 \text{ nm (a frequency-doubled Nd:YAG laser), where we have used } n_0 = 2.7 \) [27].

In order to get the exact analytical solutions for Eq. (3), we introduce the following transformations of function and variables:

\[ U(X, Y, Z) = \frac{1}{W(Z)} \Phi(\xi_1, \xi_2, \eta) e^{i \theta(X, Y, Z)}, \]

\[ \xi_1 = \frac{X - X_c(Z)}{W(Z)}, \quad \xi_2 = \frac{Y - Y_c(Z)}{W(Z)}, \quad \eta = \eta(Z), \]

where \( W(Z) \) and \( X_c(Z), Y_c(Z) \) represent the width and the central positions of the beam, respectively, all of which are the real functions of \( Z \), and

\[
\theta(X, Y, Z) = A(Z) + B_1(Z) X + B_2(Z) Y + C(Z) (X^2 + Y^2) \quad \text{(6)}
\]
denotes the phase, where \( A(Z), B_1(Z), B_2(Z), \) and \( C(Z) \) are the parameters related to the phase offset, the frequency shift, and the phase-front curvature, respectively. Substituting transformations (4)–(6) into Eq. (3) and demanding that

\[ \frac{d^2 W}{dZ^2} = FW - \frac{K}{W^3}, \]

\[ \frac{d \eta}{dZ} = \frac{1}{W}, \quad C = \frac{1}{2W} \frac{dW}{dZ}, \]

\[ \frac{d X_c}{dZ} = B_1 + 2CX_c, \quad \frac{d B_1}{dZ} = -2CB_1 + \frac{KKX_c}{W^4}, \]

\[ \frac{d Y_c}{dZ} = B_2 + 2CY_c, \quad \frac{d B_2}{dZ} = -2CB_2 + \frac{KKY_c}{W^4}, \]

\[ \frac{dA}{dZ} = - \frac{K}{2W^2} (X_c^2 + Y_c^2) - \frac{1}{2} (B_1^2 + B_2^2), \]

where \( K \) is the constant, then Eq. (3) can be reduced to the following \((2+1)\)-dimensional NLS equation with constant coefficients

\[
\frac{i}{\partial \eta} \frac{\partial \Phi}{\partial \eta} + \frac{1}{2} \nabla_{\xi_1, \xi_2}^2 \Phi + \frac{K}{2} (\xi_1^2 + \xi_2^2) \Phi + \sigma |\Phi|^2 \Phi = 0, \quad \text{(12)}
\]

where \( \nabla_{\xi_1, \xi_2}^2 = \partial^2 / \partial \xi_1^2 + \partial^2 / \partial \xi_2^2 \). Note that we have required the following integrability condition:

\[ G = - \frac{1}{R} \frac{dR}{dZ}. \quad \text{(13)}
\]

Note that the difference between the original Eqs. (3) and (12) is that the latter possesses the \( Z \)-independent nonlinearity and the harmonic potential. Thus we can obtain information on the original Eq. (3) by investigating Eq. (12) under integrability condition (13).

It should be pointed out that integrability condition (13) differs from the counterpart of \((1+1)\)-dimensional case [31], where the integrability condition is dependent on the beam width \( W \), which leads to the new discovery of the self-similar solitary solutions without the restriction on the strength of diffraction and medium nonlinearity, as shown in Ref. [7].

In this work, we consider the \((2+1)\)-dimensional case. In the particular case when the nonlinear parameter \( R \) is a constant, the gain or loss function \( G \) equal to zero. In this situation, if \( K = 0 \), Eq. (12) is reduced to standard \((2+1)\)-dimensional NLS equation by which Manassah [40] studied the collapse of the two-dimensional spatial soliton in a parabolic-index material with self-focusing Kerr nonlinearity \((\sigma = 1)\). Here we present a more general formula and are interested in the case where the nonlinear parameter \( R(Z) \) being the function of \( Z \) and \( K \) being nonvanishing, which can
be used to discuss the properties of dark stripes on the paraboloidal background for Eq. (3) with the aid of Eq. (12).

III. SELF-SIMILAR SOLUTIONS WITH PARABOLOIDAL ENVELOPE

We consider a localized stationary solution of Eq. (12) in the form \( \Phi(\xi_1, \xi_2, \eta) = \Psi(\xi_1, \xi_2) \exp(-i \beta \eta) \), where \( \beta \) is the propagation constant and \( \chi(\xi_1, \xi_2) \) is the real function. Thus, Eq. (12) can be written as

\[
\frac{1}{2} \nabla^2 \chi + \beta \chi + \frac{K}{2} (\xi_1^2 + \xi_2^2) \chi + \alpha \chi^3 = 0.
\]  

(14)

In general, the profile of the localized stationary solution for Eq. (14) can be found numerically. However, there are two cases for which the exact or approximate solution can be easily obtained. When the energy of optical beam is weak, the nonlinearity is negligible, and \( \Psi(\xi_1, \xi_2) \) can be described by the Hermite-Gaussian function. On the contrary, when the energy of optical beam is high, the diffraction term is negligible, and the profile of the stationary solution can be approximately presented by the compact paraboloidal function. In this case, the localization of the solution requires \( \sigma \beta < 0 \) and \( K \beta > 0 \), which imply that \( \beta/|K| \) is large enough. Thus, we only consider the case of \( \sigma = -1 \), and the approximate compact paraboloidal solution can be written as

\[
\Psi(\xi_1, \xi_2) = \begin{cases} 
\frac{\sqrt{\beta - \frac{1}{2}(\xi_1^2 + \xi_2^2)}}{0}, & \xi_1^2 + \xi_2^2 \leq 2\beta \\text{and} \xi_1^2 + \xi_2^2 > 2\beta,
\end{cases}
\]  

(15)

which corresponds to the well-known Thomas-Fermi approximation [41], where \( \beta > 0 \) is the propagation constant. Here for the sake of simplicity, we have taken \( K = -1 \). Thus in this situation, the efficient width of the paraboloidal beam given by Eq. (4) is \( \sqrt{2\beta W^2(Z)} \), and the corresponding total energy is of the form \( P_0(Z) = \int \int |U(X, Y, Z)|^2 dX dY = \pi \beta^2 / R(Z) \), which is independent of the beam width \( W \).

To study the evolution behavior of dark soliton stripe on the paraboloidal background for Eq. (3), we first perform the evolution of the approximate stationary solution with the paraboloidal type. We assume that the width of the paraboloidal beam keeps unchanged. Thus we consider a special situation with \( F = F_0 \) (which is constant). In this case, one can get directly the beam width \( W(Z) = (F_0)^{-1/4} \) from Eq. (7), where \( F_0 < 0 \) corresponds to the linear focusing lens effect. Thus from Eqs. (8)–(11), one can obtain \( \eta = Z/W^2, C = 0, \) and

\[
B_1 = A_{01} \cos(\sqrt{|F_0|} Z) + B_{01} \sin(\sqrt{|F_0|} Z),
\]

\[
B_2 = A_{02} \cos(\sqrt{|F_0|} Z) + B_{02} \sin(\sqrt{|F_0|} Z),
\]

\[
X_c = \frac{A_{01}}{\sqrt{|F_0|}} \sin(\sqrt{|F_0|} Z) - \frac{B_{01}}{\sqrt{|F_0|}} \cos(\sqrt{|F_0|} Z),
\]

which can be used to investigate nonlinear tunneling of solitons through the nonlinear barrier (or well) depending on the value of the parameter \( \hbar (-1 < h < 0) \) for well and \( \hbar > 0 \) for barrier [44,45]. Here the parameters \( \delta, Z_0, h \) are related to the width, the longitudinal location, and the height of nonlinear barrier or well, respectively. In this situation, the gain (loss) function is of the form

\[
G(Z) = 1 + h \sech^2(\delta(Z - Z_0)),
\]

(16)

where \( R(Z) = 1 + h \sech^2(\delta(Z - Z_0)) \).

Figure 1 presents the evolution of a paraboloidal beam for the situation given by Eq. (16). It can be seen that the paraboloidal beam can pass similarly through the nonlinear well (\( -1 < h < 0 \)) and propagate stably beyond a hundred diffraction length. The same phenomenon appears in cases \( h > 0 \) and \( h = 0 \), as shown in Fig. 2, from which one can see that the paraboloidal beam can pass similarly through the nonlinear well or barrier, and undergoes the self-similar amplification or reduction at \( Z = Z_0 \) in the total energy and the peak power.

IV. DYNAMICS OF DARK SOLITON STRIPES ON THE PARABOLOIDAL BACKGROUND

Based on this stable paraboloidal beam, in this section, we will investigate the dynamics of dark soliton stripe on the paraboloidal background. It should be pointed out that, math-
Mathematically, dark spatial solitons are particular solutions of the NLS equation with the defocusing nonlinearity. Although (2+1)-dimensional exact dark soliton solution is not known, considerable experimental and numerical results have shown that it possesses a number of characteristics in common with dark soliton solutions for the integrable (1+1)-dimensional NLS equation, and rich and more interesting phenomena, which are different from (1+1)-dimensional case, can happen in (2+1)-dimensional case [27,35–37].

Thus in order to address the properties of fundamental and higher-order dark soliton stripes on the paraboloidal background, we introduce the function

$$U(X,Y,0) = \left(-\frac{F_0}{\sqrt{R(0)}}\right)^{1/4}N\Psi(\xi_1,\xi_2)\tanh(\sqrt{2}z_1)e^{i\theta(X,Y,0)}$$

as an initial beam, where $N$ represents the order of solitons and $\Psi(\xi_1,\xi_2)$ is given by Eq. (15) with $\xi_1 = [X-X_0(Z)]/W(Z)$ and $\xi_2 = [Y-Y_0(Z)]/W(Z)$. Thus, the efficient width of the initial background beam is $\sqrt{2}\beta(-F_0)^{-1/4}$. Note that by using phase mask technique, such an input beam can introduce an $\pi$-phase jump at the center of the paraboloidal beam along the $Y$ direction, forming a dark stripe on the paraboloidal background [27,35]. The dynamic scenarios of the fundamental dark soliton stripe ($N=1$) on the paraboloidal background for the different parameter $F_0$ are shown in Fig. 3, from which one can see that the fundamental dark soliton stripe on the paraboloidal background can propagate in a stable way over large distance for $F_0=-0.01$ [see Fig. 3(a)–3(c)]. While with an increase in $|F_0|$, for example, taking $F_0=-1$, the dark soliton stripe on the paraboloidal background can only propagate for short distance (for the parameters we chose, this distance is about $Z=35$) and eventually fade away with increasing the propagation distance [see Fig. 3(d)–3(f)]. This property is different from that of the (1+1)-dimensional case [31]. Indeed, for (2+1)-dimensional case, the parameter $F_0$ only influences the width of the initial background beam and is independent of the initial total energy (which only depends on the propagation constant $\beta$), so for the given parameter $\beta$, the increase in $|F_0|$ leads to the decrease in the width of the paraboloidal background and the increase in the peak power and eventually results in the disappearance of the dark soliton stripe on the paraboloidal background due to the interaction.

Furthermore, we also performed the dynamic scenarios of high-order dark soliton stripe on the paraboloidal background. As shown in Fig. 4 for $N=2$, one can see that the paraboloidal background evolves in a periodic fashion, in which the period is about $Z=32$ [see Figs. 4(a) and 4(d)–4(f)], and a pair of gray soliton stripes appear [see Figs. 4(b) and 4(c)], just like that in Ref. [31]. However, because of the transverse perturbation, the gray soliton stripes tend to bend and after the several period eventually leads to the presence of the several vortices, as shown in Fig. 4(f).

It should be pointed out that, in Figs. 3 and 4, we take $h=0$, which corresponds to the homogenous nonlinear media, as an example to perform the dynamics of the fundamental and high-order dark soliton stripe on the paraboloidal background.
background. In fact, for the inhomogeneous nonlinear media, such as \( h = -0.4 \) and 2, the main features of the dark soliton stripes on the paraboloidal background are similar with their counterparts in homogeneous nonlinear media as shown in Figs. 3 and 4, namely, the dark soliton strips can self-similarly pass through nonlinear well and barrier. Thus our results may suggest a better way to control optical beams.

We also performed the evolution of dark soliton stripe on the paraboloidal background with an initial out-of-phase dislocation at \( \xi_1 = \xi_0 \), in which the input beam takes the form of

\[
U(X, Y, 0) = \left( -\frac{F_0}{\sqrt{R(0)}} \right)^{1/4} \Psi(\xi_1, \xi_2) \tanh(\sqrt{2}\beta(\xi_1 - \xi_0)) \exp[i\theta(X, Y, 0)],
\]

where \( \xi_0 \) is apart from the boundary of the paraboloidal background \( \xi_1 = \sqrt{2}\beta \). As shown in Fig. 5, it is clearly seen that the dark soliton stripe initially moves from right to left on the paraboloidal background. However, the dark stripe becomes bending due to the different transverse velocity which is induced by the different intensity of background along \( X \) direction, and eventually the dark soliton stripe breaks into the two vortex solitons that oscillate periodically on the background [see Figs. 5(d)–5(f)]. This is clearly different from the case of Figs. 3(a)–3(c). So we can infer that the dark stripe deviated from the central position of the paraboloidal background may induce the generation of the optical vortex.

V. INTERACTION OF DARK SOLITON STRIPES ON THE PARABOLOIDAL BACKGROUND

In this section, we will discuss the interaction of dark soliton stripes on the paraboloidal background. Figure 6 shows the dynamic scenarios of two and three parallel dark soliton stripes on the paraboloidal background. From Fig. 6, one can see that two (the first row) and three (the second row) parallel dark soliton stripes on the paraboloidal background experience elastic collisions within a certain distance. The considerable numerical simulations show that such parallel dark soliton stripes on the paraboloidal background pos-
FIG. 6. (Color online) The interaction of two and three parallel dark soliton stripes with the initial separations 4 and 3, respectively, on the paraboloidal background for system (16) with $h=0$, $\delta=0$, and $Z_0=25$. Here the first row corresponds to the evolution of two parallel dark soliton stripes with $Z=(a)\ 0$, (b) 18 (b), and (c) 36 and the second row is the evolution of three parallel dark soliton stripes with $Z=(d)\ 0$, (e) 18, and (f) 36, where $F_0=−0.01$ and other parameters are the same as in Fig. 3.

FIG. 7. (Color online) The interaction of two parallel dark soliton stripes with the initial separation 4 on the paraboloidal background for system (16) with $h=0$, $\delta=0$, and $Z_0=25$. Here $Z=(a)\ 0$, (b) 17, (c) 34, (d) 55, (e) 65, (f) 81, and (g) 100, where other parameters are the same as in Fig. 6 except for $\beta=10$.

FIG. 8. (Color online) The dynamics of two cross dark soliton stripes on the paraboloidal background for system (16) with $h=0$, $\delta=0$, and $Z_0=25$. Here $Z=(a)\ 0$, (b) 50, and (c) 100, where $F_0=−0.01$ and other parameters are same as in Fig. 3.
We should also note that it will be absolutely different when the total energy and the peak power of the background beam get much higher, which depends on the choice of the propagation constant $\beta$, such as $\beta=10$. Figure 7 presents the patterns of the interaction of two parallel dark stripes on the paraboloidal background for the propagation constant $\beta=10$. From it, one can see that after a period, two parallel dark stripes evolve into six pair vortex solitons [see Fig. 7(c)]. Successively, a variety of patterns appears due to their interactions [Figs. 7(d)–7(g)]. Here, Fig. 7(h) is the phase profile of Fig. 7(f). From Fig. 7(f), one can see that eight vortex solitons are symmetrically distributed on the paraboloidal background.

Finally, we perform the interaction of cross dark stripes on the paraboloidal background. Figure 8 presents the dynamic scenarios of two mutually orthogonal dark stripes on the paraboloidal background. From Fig. 8, one can find that the two mutually orthogonal dark stripes on the paraboloidal background can propagate stably. Thus our results suggest an efficient way to generate necklace-ring soliton [46]. Thanks to complexity of their interaction, the patterns of the interaction of four cross dark stripes on the paraboloidal background, and they eventually evolve into the patterns of symmetrical distribution on paraboloidal background [Fig. 9].

FIG. 9. (Color online) The dynamics of four cross dark soliton stripes with the initial separation 4 on the paraboloidal background for system (16) with $\eta=0$, $\delta=0$, and $Z_d=25$. Here $Z=(a)$ 0, (b) 20, (c) 32, (d) 56, and (e) 100, where other parameters are the same as in Fig. 8 excepted for $\beta=10$.

VI. CONCLUSIONS

We have demonstrated the self-similar evolution of dark soliton stripes on the paraboloidal background in a bulk graded-index waveguide with self-defocusing nonlinearity. It has been shown that under proper parameter condition the fundamental dark soliton stripe on the paraboloidal background can propagate stably for more than hundreds of diffraction length. Our results also showed that the higher-order dark soliton stripe on the paraboloidal background is less stable. Furthermore, we have investigated the interactions of dark soliton stripes on the paraboloidal background and found some phenomena that are different from the (1+1)-dimensional case discussed in Ref. [31]. Due to the robustness of dark soliton stripes on the paraboloidal background, we believe that our results will be useful for the beam steering, all-optical switch, and optical encoding.

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