Nonlinear localized modes at phase slips in two-dimensional photonic lattices

Mario I. Molina
Departamento de Física, Facultad de Ciencias, Universidad de Chile, Las Palmeras 3425, 7800024 Nunoa, Santiago, Chile

Yuri S. Kivshar
Nonlinear Physics Center, Research School of Physics and Engineering, Australian National University, Canberra, Australian Capital Territory 0200, Australia
(Received 14 September 2009; published 4 December 2009)

We analyze the existence and stability of nonlinear modes localized at phase-slip waveguide channels and their intersections in two-dimensional photonic lattices with rectangular symmetry. While in linear lattices only one localized mode of this type may exist under special conditions, nonlinearity supports a variety of localized modes including the modes which bifurcate from the symmetric states and describe nonlinear asymmetric localized states. We also study the surface modes localized at the waveguide channel edges in such lattices.

DOI: 10.1103/PhysRevA.80.063812 PACS number(s): 42.65.Tg, 42.65.Wi

I. INTRODUCTION

The study of nonlinear dynamics in waveguide arrays and photonic lattices has attracted special attention due to a possibility to observe many novel effects of nonlinear physics and suggest interesting applications in photonics [1]. In particular, it was shown that discrete nonlinear photonic systems can support different types of spatially localized states in the form of discrete solitons [1,2]. These solitons can be controlled by the insertion of suitable defects in an array, as was suggested theoretically [3,4] and also verified experimentally for arrays of optical waveguides [5]. Defects may provide an additional physical mechanism for light confinement, and they can support both linear and nonlinear localized modes, which have been studied theoretically for different linear and nonlinear models [6–10] and observed experimentally in one-dimensional photonic lattices [6,11].

Recently, a type of nonlinear defect mode has been presented [12] at the interface between two similar nonlinear one-dimensional waveguide arrays. The defect modes described for that system are somewhat linked to the experimental observation of linear localized modes at the interface of different AlGaAs waveguide arrays [13]. In particular, it was demonstrated that the nonlinear phase-slip defect modes possess the specific properties of both nonlinear surface modes and discrete solitons. In this paper, we extend substantially the earlier results [12] and analyze the existence and stability of nonlinear modes localized at phase-slip waveguide channels and their intersections in two-dimensional photonic lattices with rectangular symmetry. This geometry and the modes are similar to the recently discussed problem of phase-slip defects in two-dimensional photonic crystals [14,15]. The increase in dimensionality brings new interesting cases to be examined, such as the interplay among the increased spatial coupling, the presence of an extended defect, the possibility of anisotropy, and the presence of a bona fide surface. We demonstrate that, while in linear lattices analyzed previously only one localized mode of this type may exist under special conditions, nonlinearity supports a variety of localized modes including those that appear through the symmetry-breaking bifurcations and associated with nonlinear asymmetric localized modes. We also study the surface modes localized at the waveguide channel edges which generalize nonlinear surface modes [16,17] in waveguide arrays. We also analyze stability of the localized modes and their generation in both linear and nonlinear regimes.

The paper is organized as follows. In Sec. II we present our model. In Sec. III we obtain exact results for linear guided modes supported by phase-slip waveguide channels and an intersection of two such channels in a rectangular photonic lattice. Then, in Sec. IV we analyze nonlinear localized modes in a single phase-slip waveguide channel and describe the symmetry-breaking instability and asymmetric localized modes. Section V is devoted to the study of nonlinear localized modes at the intersection of two phase-slip channels and their dynamical generation, whereas Sec. VI discusses surface modes. Finally, Sec. VII concludes the paper.

II. MODEL

We consider a two-dimensional photonic lattice of nonlinear optical waveguides, as shown in Figs. 1(a) and 1(b). We create a waveguide channel in such a lattice by breaking its translational symmetry in two ways. First, we consider a waveguide channel created by a shift of a half of the lattice, so that the coupling between two neighboring rows of waveguide is changed [see Fig. 1(a)]. Second, we create an inter-
section of two such phase-slip waveguides, as shown schematically in Fig. 1(b), and discussed earlier for photonic crystals [14,15]. In both of these cases, the interaction of the waveguides along the phase-slip channel is changed, and two mismatched waveguides interact with a different coupling strength, $V' \neq V$, where $V$ is the coupling between the waveguides in a perfect lattice.

We describe such phase-slip waveguides by employing the coupled-mode theory that allows deriving a system of discrete equations for the normalized amplitudes $E_n$,

$$\frac{i}{2} \frac{dE_n}{dz} + \sum_q V_{n,q} E_q + \gamma |E_n|^2 E_n = 0,$$  \hspace{1cm} (1)

where $\gamma = 1$ ($-1$) denotes focusing (defocusing) nonlinearity, $n$ is a two-dimensional index, $n = (n_x, n_y)$, and $V_{n,q}$ is the matrix of the coupling coefficients, which is only nonzero when $n$, $q$ are nearest-neighbor sites, in which case $V_{n,q} = V$, except for the coupling across the phase slip where $V_{n,q} = V'$. The amplitude $E_n$ is defined in terms of the actual electric fields $E_n = (2\pi \eta_0 n_0 \lambda^2)^{1/2} E_n^0$, where $\eta_0$ is the free-space wavelength, $\eta_0$ is the free-space impedance, $n_2$ and $n_0$ are nonlinear and linear refractive indices of each waveguide, and $\gamma = \pm 1$ defines the character of the nonlinear response (focusing or defocusing).

We are interested in the stationary localized modes of the form, $E_n = \exp(i\beta z) C_n$, where $\beta$ is the propagation constant, and the mode amplitude $C_n$ satisfies the stationary equations,

$$\beta C_n + \sum_q V_{n,q} C_q + \gamma |C_n|^2 C_n = 0.$$  \hspace{1cm} (2)

It is worth noticing that Eq. (2), with the $V_{n,q}$ defined above possesses staggered-unstaggered symmetry. The transformation $\beta \rightarrow -\beta$, $\gamma \rightarrow -\gamma$, and $C_n \rightarrow (-1)^{n_x n_y} C_n$ leaves Eq. (2) invariant. The proof is as follows: after performing the transformation on Eq. (2) for an arbitrary $V_{n,q}$, one obtains

$$\beta C_n + \sum_q (-1)^{n_x n_y} V_{n,q} C_q + \gamma |C_n|^2 C_n = 0,$$  \hspace{1cm} (3)

which reduces back to Eq. (2) for all $V_{n,q}$ that couple nearest-neighbor sites only, independently of the actual (nonzero) value of $V_{n,q}$. Other possible, albeit less physically relevant cases, include coupling to third- or fifth-nearest neighbors only. In this work, our choice of $V_{n,q}$ is a special case that obeys the staggered-unstaggered symmetry. Hereafter, we consider only $\beta > 0$ and $\gamma = 1$.

In our analysis, we consider a two-dimensional square lattice of $22 \times 22$ waveguides. To visualize the field in the lattice, we present the field as $U(x,y) = \sum_{m \pm 1} C_{n,m} \phi(x-n,y-m)$, where $\phi$ is a guided mode of a single waveguide centered at the site $(n,m)$. For the latter function, we assume a generic form, $\phi(x,y) = \exp[-(x^2+y^2)/\sigma^2]$, taking $\sigma = 0.1$.

III. LINEAR LOCALIZED MODES

First, we analyze the existence of guided modes in the linear regime when $\gamma = 0$, and the stationary modes are described by the equation

$$-\beta C_n + \sum_q V_{n,q} C_q = 0.$$  \hspace{1cm} (4)

For the single phase-slip channel localized between $n = 0$ and $n = 1$ waveguides, we look for the mode profile in the form, $C_{n,m} = A \xi^{n-m}$, for $n > 1$, and $C_{n,m} = A \xi^n$, for $n < 0$, and obtain

$$\xi = (V/V'),$$  \hspace{1cm} (5)

and

$$\beta = V \left(2 + \frac{V}{V'} + \frac{V'}{V} \right).$$  \hspace{1cm} (6)

The results obtained above describe a linear mode guided by a single phase-slip channel that is unbounded in the propagation direction being localized across the channel. An example of the profile of such a mode is shown in Fig. 2(a) for a $22 \times 22$ lattice and $V'/V = 2$, where the mode is actually localized longitudinally due to the finite size of the lattice.

To obtain the results for the crossing phase-slip channels we assume that the cross is located between the waveguides with the indices $n = 0$ and $m = 0$. We look for the localized mode in the form, $C_{n,m} = \xi^{n+m-2}$, for $n, m > 1$, $C_{n,m} = \xi^{-n-m}$, for $n, m < 0$; $C_{n,m} = \xi^{n-1-m}$, for $n > 1$ and $m < 0$, and $C_{n,m} = \xi^{-n+m-1}$, for $n < 0$ and $m > 1$. Substituting this ansatz into the linear Eq. (4), we obtain

$$\xi = (V/V'),$$  \hspace{1cm} (7)

and

$$\beta = 2V \left(\frac{V}{V'} + \frac{V'}{V} \right).$$  \hspace{1cm} (8)

An example of its spatial profile for a $22 \times 22$ lattice and $V'/V = 2$ is shown in Fig. 2(b).

It is clear from Eqs. (5) and (7) that, in order to have a localized mode in both, the linear lattice with a phase-slip channel and crossing phase-slip channels, we should satisfy the condition $V'/V > 0$. This result was found earlier for the channel crossing [14]. From exact expressions (6) and (8), the assumption $V'/V > 0$, and the use of the well-known identity $|x + (1/x)| \geq 2$, it is easy to prove that the propagation constant for the mode localized at the waveguide crossing is always larger than for the mode localized at the phase-slip channel. They are equal only when $V' = V$ and both propagation constants touch the linear band.

IV. SINGLE PHASE-SLIP CHANNEL

Next, we consider the nonlinear case described by Eqs. (1) and (2) for $\gamma = 1$. For a given value of $\beta$, the system of the
stationary Eq. (2) is solved numerically by a multidimensional Newton-Raphson scheme. As we are interested in the modes localized near the phase-slip channel, we look for the modes with the mode maxima near the slip boundary that decay quickly across the channel. A standard linear stability analysis even mode (0,0) for β=6.5, (b) symmetric even (+,+), mode for β=6.4, (c) longitudinal even (−) mode for β=6.5, (d) twisted (+,−) mode for β=5.5. In all cases, γ=1, V'/V=2. White lines mark the position of the waveguide channel located between 11<x<12.

FIG. 3. (Color online) Examples of the nonlinear localized modes guided by a single phase-slip channel: (a) asymmetric odd (+,0) mode for β=6.5, (b) symmetric even (+,+), mode for β=6.4, (c) longitudinal even (−), mode for β=6.5, (d) twisted (+,−), mode for β=5.5. In all cases, γ=1, V'/V=2. White lines mark the position of the waveguide channel located between 11<x<12.

comes unstable above a certain threshold power, and it transforms into an odd mode centered at any of the two equivalent sites coupled by the bond impurity [see Fig. 4(a)]. This result can be easily understood from the known instability of an even mode for a discrete homogeneous lattice: As the power is increased, the effective coupling is decreased and the distinction between V and V' becomes blurred. In the high-power limit, the evenlike localized state “feels” as inside an homogeneous lattice; hence the onset of instability. On the contrary, for V'<V, the sites close to the channel “feel” as if they were close to a surface, and consequently, the power curves begin to resemble the ones for a surface mode. In particular, a minimum power is now required to “anchor” any localized mode in the immediate vicinity of the channel.

V. CROSSING OF TWO CHANNELS

Now we consider the case when two of the previous waveguide channels intersect each other in a perpendicular manner, forming a cross channel [see Fig. 1(b)]. This reduction in symmetry will have an impact in the low-lying nonlinear modes, but it will not affect the higher-order ones, because of the anticontinuum limit. As above, we look for stationary localized modes centered in the vicinity of the channels intersection. Figure 5 displays several low-order nonlinear modes. Some of them were also found for the single-channel configurations, like the asymmetric odd mode [Fig. 5(a)], the symmetric even mode [Fig. 5(b)], and the even-even mode [Fig. 5(c)]. But now we have a number of new modes, such as the diagonal odd-odd mode [Fig. 5(d)], the diagonal dipole mode [Fig. 5(e)], and the quadrupolelike mode [Fig. 5(f)]. Figure 6 shows the power vs propagation
constant curves for these modes, for the cases $V'/V>1$ and $V'/V<1$, which differ qualitatively.

The dynamical excitation of these modes can be accomplished in principle, from the evolution of a judiciously chosen initial beam profile with the right symmetry and power level. In order to diminish the effects of backscattered radiation from the finite lattice boundaries, we have endowed the boundary lattice sites with a phenomenological loss term of the form $\epsilon E_0$, in Eq. (1), with $\epsilon \sim O(1)$. In Figs. 7 and 8 we show some examples, where an initially localized mode is evolved in space according to Eq. (1). If we start from a high-power, completely localized, evenlike configuration $^{(0)}_{(+)}$ and $V'>V$ then, due to the instability of the even mode (Fig. 6), the state should decay to a single-site odd mode, after shedding some power via radiation. This is clearly shown in Fig. 7. Next, and also for $V'>V$, we excite a low-power localized even-even-like state $^{(0)}_{(0)}$. Since the even-even mode is always stable, the state sheds the extra optical power, producing a final stable even-even configuration, shown in Fig. 8. On the contrary, under the same initial conditions but taking $V'/V=1/2$, the initial state decays entirely into radiation. If we now increase the amount of power into the initial even-even initial configuration, it decays into the first available stable mode: the twisted mode. All of these results are fully consistent with the two power diagrams presented on Fig. 6.

VI. SURFACE MODES

Existence of novel types of discrete surface solitons localized in the corners or at the edges of two-dimensional photonic lattices [19–21] have been recently confirmed by the experimental observation of two-dimensional surface solitons in optically induced photonic lattices [22] and two-dimensional laser-written waveguide arrays in fused silica [23,24]. These two-dimensional nonlinear surface modes demonstrate novel features in comparison with their counterparts in truncated one-dimensional waveguide arrays [16,17,25]. In particular, in a sharp contrast to one-dimensional discrete surface solitons, the mode threshold is lower at the surface than in a bulk making the mode excitation easier [20].

Here, we employ our model of a phase-slip channel and study localization of light at the edge of the lattice, in the
vicinity of the phase slip. We reveal that the effectively one-dimensional nature of the waveguide created in a two-dimensional lattice leads to the localized surface modes which resemble the properties of the discrete surface solitons in waveguide arrays [16,17,25].

Figures 9(a)–9(c) show examples of a low-power nonlinear surface modes, which do not have linear counterpart and require a threshold power for their excitation. The lowest mode (a) is asymmetric [see Fig. 9(b)] and it is localized predominantly on one of the sides of the slip-phase waveguide channel. The symmetric mode (c) has a higher power, whereas other modes, including the twisted mode, correspond to higher branches in Fig. 9(a). The stability analysis of the surface nonlinear modes shows that the well-known Vakhitov-Kolokolov stability criterion [2] seems to hold so that the lowest branch with a positive slope in Fig. 9(a) describes stable surface modes.

VII. CONCLUSIONS

We have presented and analyzed types of nonlinear guided modes localized at phase-slip channels and their intersections in two-dimensional photonic lattices with rectangular symmetry, extending in this way a previous work on one-dimensional arrays [12]. While in a linear lattice with such a phase-slip channel only one localized mode of this type may exist under special conditions, nonlinearity can support a variety of localized modes including the modes which bifurcate from the symmetric guided modes and describes nonlinear asymmetric localized states. We have also studied the surface modes localized at the channel edges with the properties resembling those of surface solitons, and demonstrated that all such localized modes can be generated in both linear and nonlinear regimes.

ACKNOWLEDGMENTS

This work was supported by Fondecyt (Grant No. 1080374) in Chile and by the Australian Research Council in Australia. Y.K. thanks Departamento de Física at Facultad de Ciencias (Universidad de Chile) for hospitality.