Generation of high-order optical vortices by optical wedges system

Ya.V. Izdebskaya, V.G. Shvedov, A.V. Volyar

National Taurida V Vernadsky University,
Vernadsky av. 4, Simferopol, 95007, Crimea, Ukraine
tel. (0652)23-02-48, e-mail: volyar@ccssu.crimea.ua

ABSTRACT

The aim of the given report is experimental and theoretical research of the diffraction of a Gaussian beam by the optical wedges system. It is shown that this system is able to form high-order optical vortices. The effectiveness of the system is about 90%. It was shown that each wedge changes a charge of phase singularity as a result of edge diffraction. The value topological charge of the optical vortex formed after system is defined by the number of wedges in the system. Changing mutual orientation corners of wedges we can select required conditions of the vortex core. It was revealed that the optical vortex appears structurally steady if the corner of mutual orientation of wedges equals $\alpha = \pi/n$ (where $n$-number of wedges).

Keywords: optical vortex, dielectric-wedge system

1. INTRODUCTION

One of the highlights in modern research is the processes of generation of high-order singular beams. Such beams are widely used in optoelectronics. They are also capable of capturing microparticles due to their own orbital moment.

To present day the creation of high-order optical vortices was based on the phase-transparency technique [1], the computer-generated holograms [2] and the polarized transformations in uniaxial crystals [3]. There existed a generally accepted belief that high-order beams are structurally unstable field formations, which means that in the process of propagation a high-order optical vortex is to disintegrate into a number of single optical vortices. The vortex disintegration process is influenced both by the beam propagation in itself and the diffraction on any weak heterogeneities of the wave propagation medium.

However, the method of obtaining singular beams on the edge of the dielectric wedge [4] demonstrated by us not long ago revealed a possibility of obtaining singular beams of higher order, if the wedges should be located consecutively one after another (Fig. 1). Such beams possess a number of specific properties, which make them distinguishable from the Laguerre- and Hermite-Gaussian beams. The key property of such beams is structural stability.

The aim of the given paper is to study the physical mechanism of structural stability high-order optical vortices by the dielectric-wedge system.

2. THEORY

Let the fundamental Gaussian beam with the wavefunction:

$$\Psi_0(r, z) = \frac{\exp(-ikz)}{\sigma} \exp \left(-\frac{r^2}{\rho^2\sigma}\right)$$

(1)

(where $r^2 = x^2 + y^2$, $z_0 = k\rho^2/2$ is the Rayleigh length, $\sigma = 1 + iz/2z_0$, $\rho$ is the waist radius at $z=0$) is diffracted by the edge of the dielectric wedge with the refractive index:

$$n(x) = \begin{cases} n_\ast, & x > 0, \\ 1, & x \leq 0. \end{cases}$$

(2)

where $n_\ast$ is the refractive index of the transparency.
The wave function of the beam at the wedge plane \(z=0\) is
\[
\Psi_w(x,y,0) = \Psi_{00}(x,y,0) \exp\left[i ky [n(x) - 1] \times \tan \alpha + i k d\right]
\]
where \(d\) is the wedge thickness at the beam axis:
\[
d = [(n(x) - 1)[2m + 1] \frac{d}{2}, \ m = 0, 1, 2, 3, \ldots]
\]
As was shown in the article [4] the wavefunction of the beam in a far diffraction zone has the form:
\[
\Psi(x,y,z) = \frac{i \pi z}{2k \sigma} \psi_0(x,y,z) \left\{ \exp(i ky \tan \beta) \times \exp(i \theta) \text{erfc} \left[ \frac{-ikx}{\sqrt{2z \sigma}} \right] + \text{erfc} \left[ \frac{ikx}{\sqrt{2z \sigma}} \right] \right\}
\]
where \(\theta = kd\) is the additional phase of the wave transmitting through the wedge substrate.

The diffraction process presented by eq. (5) enables us to transform the smooth Gaussian beam into the singular beam with the chain of optical vortices. In order to select the single optical vortex from the vortex chain it is necessary to perform the phase matching conditions [5]:
\[
\left( \frac{2}{\sqrt{\pi}} + X \right) \exp(-X^2) = \frac{2}{\sqrt{\pi}}
\]
where \(X = \sqrt{kz_0} \Lambda; \ \Lambda = (n - 1) \tan \alpha\) so that the thickness of the wedge substrate satisfies the requirement:
\[
h = m \lambda, \ m = 1, 2, 3, \ldots
\]
Let the beam now transmits through two wedges (Fig. 1). The wedges have the same angles of slope \(\beta_1\) and \(\beta_2\) and also the transparencies make the angle \(\alpha\) between each other. The beam transformed by the system of optical wedges can be presented as:
\[
\Psi_2(x,y,z) = \frac{1}{kz} \int dX' \psi'_2(x', y', 0) \exp\left[-i \frac{k}{2} \left(\frac{x - x'}{z} + \frac{y - y'}{z}\right)\right] \times \exp\left(i k \cdot \left(\frac{d_z n(x') - z}\right) \exp\left(iky' n(x') \tan(\beta_2)\right)\right)
\]
where \(d_z\) and \(d_\perp\) - the thickness of the first and second wedges at the beam axis, \((x', y') = 0, z = 0\) stands for the boundary of the second wedge, so that:
\[
\begin{align*}
x' &= x' \cos(\alpha) - y' \sin(\alpha) \\
y' &= y' \cos(\alpha) + x' \sin(\alpha)
\end{align*}
\]

The presence of the second wedge changes the phase matching condition (7). The thickness \(d_\perp\) of each wedge at the beam axis have to satisfy the requirement:
\[ d_i = \frac{\lambda (2m+1)}{2(n_n - 1)}, \quad m=0,1,2,\ldots \]  

But for all that the phase difference between the part of the beam transmitting through the wedge and the beam spreading in air must be:

\[ \theta = \pi (2m + 1), \quad m = 0,1,2,\ldots \]  

Consequently, the thickness \( d_2 \) of the second wedge is:

\[ d_2 = (m+1)d_1, \quad m=0,1,\ldots \]  

Consider two cases of the mutual orientation of two wedges. Let's rotate the wedge around the axis at the angle \( \alpha \) (Fig.1):

1) The wedges are oriented in such a way that at \( \alpha=0 \) and \( \beta_1=-\beta_2 \).

In this case, all angles \( 0 \leq \alpha \leq \pi \) show no formation of screw dislocation on the beam axis, as at \( \alpha=0 \) the system forms a parallel-sided plate (\( \angle \beta = \beta_1 + \beta_2 = 0 \)).

2) The wedges are oriented in such a way that at \( \alpha=0 \) and \( \beta_1=\beta_2 \).

This case is particularly of interest, as at prescribed thickness of both wedges high-order optical vortices can be formed in the center of the beam. Let us consider this situation in a more detailed way:

a) \( \alpha = \pi m \); in this case the wave function of the beam which has passed the two-wedge system to an accuracy of constant factor can be presented as:

Fig.2. The sketch of the double-wedge system at \( \alpha=\pi \).

Fig.3. The intensity distributions in the beam diffracted by the double-wedge system: the vortex chains in the mismatched (a-theory; b-experiment) and matched (c-theory; d-experiment) systems (\( \alpha=0, \pi; \rho=3 \text{ mm}; n_n=1.5 \)).
\[ \Psi(r, z) \sim 2\Psi_0(r, z) \left\{ \text{erfc} \left( \frac{ikr}{\sqrt{ikz}} \right) \sin[ky(n-1)\gamma(g(\beta)) - \cos[ky(n-1)\gamma(g(\beta))] \right\} \]  

(13)

where \( n \) is the refractive index of plate medium.

In this case, as follows from expression (13), the number of optical vortices on the boundary of the wedges doubles, while no vortex is formed on the optical axis (Fig. 3 c,d).

b) \( 0 < \alpha < \pi \); when the angle changes on the beam axis within these limits, the screw dislocation is preserved, while the optical vortex charge doubles (Fig. 4). The expression (8) will then look like this:

\[ \Psi(r, z) = \Psi_0(r, z) \left\{ \text{erfc} \left( -\frac{\sqrt{ik} x}{\sqrt{2\sigma}} \right) \text{erfc} \left( \frac{\sqrt{ik} y_1}{\sqrt{2\sigma}} \right) - \exp(ik\tan \beta) \text{erfc} \left( -\frac{\sqrt{ik} x}{\sqrt{2z\sigma}} \right) \text{erfc} \left( -\frac{\sqrt{ik} y_1}{\sqrt{2z\sigma}} \right) \right\} \]

(14)

As far as

\[ \lim_{r \to 0} \Psi(r, z) = \frac{f(x, y)}{\sigma} \Psi_0(r, z) \exp(2\phi) \]  

(15)

\( f(x, y, \alpha, \beta_1, \beta_2) \) is responsible for the amplitude distribution in the beam), the double optical vortex is positioned at the beam axis.

Fig. 4. The intensity distributions in the beam diffracted by the double-wedge system: experimental (a,b,c) and theoretical (d,e,f) patterns in the matched system if \( \alpha = \pi/2, \rho = 3\text{mm} \) (a,d); \( \rho = 1.5\text{mm} \) (b,e); experimental (c) and theoretical (f) interference spirals.

The problem can be extended onto the \( n \) number of consecutively located wedges. The below described experiment will allow us to consider this case in a more detailed way. Therefore, the number of wedges determines the full topological charge of the singular beam. The angles of mutual orientation of the wedges assign the supplemental distance between each two neighboring plates of the wave-front. Changing the mutual orientation angles we can select the vortex core states we need.

Unlike the Laguerre- and Hermite Gaussian beams, the high-order beams generated in the dielectric wedge system are structurally stable field transformations. It is virtually impossible to achieve structural stability of the Laguerre- and Hermite Gaussian beams obtained on the computer-generated holograms due to the fact that any perturbation of the Gaussian beam profile, no matter how small it could be, results in disintegration of high-order beams into single optical vortices. Indeed, the Laguerre–Gaussian beam containing an \( l \)-order optical vortex can be presented as follows:

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\[
\Psi_2 = \Psi_{00} - \Psi_{01} = [1 - \exp(i2\Gamma) + \frac{r^2}{w^2} \exp(i2\Gamma) \exp\left(-\frac{r^2}{\rho^2 \sigma^2}\right)]
\]

where \( w^2 = \rho^2 |\sigma^2| \).

On the optical axis, the strength of the beam field differs from zero, which will result in splitting of the optical vortex with double topological charge. The value of such splitting will be increasing with the increase in the value of difference of factors \( a - b \). The above theoretical arguments are fully corroborated by experiment. Figure 4 (a, b, c) shows the splitting of the optical vortex with double topological charge obtained on the computer-generated hologram located at a small angle to the direction of the laser beam propagation. To intensify the effect of splitting of the second order vortex, a diaphragm with a 1-mm pupil diameter was put up at a distance of 10 cm. The interference pattern shown in Fig. 4, b confirms the fact of splitting of the vortex with topological charge \( l = 2 \) into two single lower-order vortices.

However, it is possible to achieve the structural stability of the high-order vortex if the third member in expression (18) becomes zero. In ordinary \( LG^{(2)} \) beams, it is impossible to achieve this condition.

On the other hand, we demonstrated that under the diffraction of light on two consecutive dielectric wedges, a beam can be formed whose wave function looks like this (Fig. 4 d-f):

\[
\Psi(r, \varphi, z) = \frac{1}{\rho^2 \sigma^2} \exp\left(-\frac{r^2}{\rho^2 \sigma^2}\right)
\]

In the proximity to phase singularity the wave function can be presented as follows:

\[
\Psi(r, \varphi, z) = \left[\left(\frac{a + b}{2}\right)^2 - \left(\frac{a - b}{2}\right)^2\right] (x + iy)^2 \frac{1}{\rho^2 \sigma^2} \exp\left(-\frac{r^2}{\rho^2 \sigma^2}\right)
\]

As follows from the expression obtained, the vortex-free member in the wave function is absent, which means that such a singular beam turns out to be structurally stable towards perturbations of difference in factors \( a - b \).

In a similar way, higher-order optical vortices can be constructed, in which case the vortex remains structurally stable, provided that the wedges should be oriented at angles divisible by \( \pi/n \), where \( n \) is the number of wedges in the system:

\[
\Psi_n(x, y, z) = (a, x + ib, y)(b, x + ia, y) \times \ldots \times (b, x + ia, y) =
\]

\[
\left[\left(\frac{a + b}{2}\right)^n (x + iy)^n - \left(\frac{a - b}{2}\right)^n (x - iy)^n\right]
\]

Each multiplier in the expression (17) is responsible for the optical vortex formed by a respective wedge in the system.

This becomes possible if complex factors \( a, b \) are connected by the following relation:

\[
a_n = (a \cos \varphi_n - ib \sin \varphi_n)
\]

\[
b_n = (b \cos \varphi_n - ia \sin \varphi_n)
\]

From this relation follows that arbitrary-order structurally stable optical vortices can be formed in the dielectric wedge system in cases where the wedges are mutually oriented at the following angles:

\[
\varphi_n = \frac{\pi(n - 1)}{n}
\]

3. EXPERIMENT

The cover glasses for a microscope turned out to have a slight slope of their planes and can be used as dielectric wedges.

We selected two glasses with the identical optical thicknesses and much the same angles \( \beta_1 \) and \( \beta_2 \). One of the wedges is rotated at the angle \( \alpha \) (Fig. 1) and the intensity distribution in the diffracted beam is looked after at a far zone (at the distance about 1 m from the wedge). The results obtained are shown in Fig. 3, 4. At first, the edges of the transparencies are slightly moved aside and their verges are parallel to each other \( (\alpha = 0) \) (Fig. 2). The system are
illuminated by the wide beam (the waist radius is about 3 mm). The diffracted beam forms two vortex chains (Fig.3a,b). After the transparencies were moved together, two chains flowed together into the single vortex chain (Fig.3c,d). The cores of the single vortices in the chain have an elliptic form. The major ellipse axes are the same directed. Besides, all vortices in the chain have the same topological charge. The rotation of one of the wedges results in transforming the topological structure of the central vortices. Its charge is doubled (Fig.4).

We also studied the conditions of formation of high-order vortices at beam diffraction in the three- and four-wedge system. In case of a triple wedge (the sides here form mutual orientation angles $\alpha_1 = \alpha_2 = \alpha_3 = \pi / 3$) a beam is formed whose intensity is distributed as shown in Fig. 5a. The three branches of the interference spiral indicate the triple topological charge of phase singularity (Fig.5c). At the same time, the system containing four wedges with angles $\alpha_1 = \alpha_2 = \alpha_3 = \pi / 4$ forms a singular beam with the quadruple topological charge (Fig. 5d).

With a view to checking the structural stability of the optical vortices thus obtained, they were let through a circular diaphragm and the intensity distribution and interference patterns at different distances from it were observed. The measurements were made at a distance of up to 20 m with no changes in the vortex core structure observed in the process of the beam propagation, as shown in Fig. 2b,c.

![Image](image_url)

Fig. 5. The experimental intensity distribution (a,b,d,e) and the interference patterns (c,f) of optical vortices with topological charges $l=3$ (a,b,c) and $l=4$ (d,e,f) (the waist of the beam is $p=3$ mm).

4. CONCLUSIONS

Therefore, we have considered both theoretically and experimentally the diffraction process on the edges of consecutively located identical optical wedges. It was shown that each wedge changes a phase singularity charge as a result of the edge diffraction. The value of the topological charge of the optical vortex formed after the system is equal to the number of wedges. The sign of the topological charge of the optical vortex is determined by the sign of the wedge plane convergence angle. Positive angle $\beta>0$ corresponds to the positive topological charge, while negative angle $\beta<0$ corresponds to the negative topological charge. It was found out that the beam-transfered vortex proves to be structurally stable, if the angle of mutual orientation of the wedges $\alpha=\pi/n$ (where $n$ is the number of wedges). If the
angle between the sides of two neighboring wedges $\alpha=0$ and $\alpha=\pi$, then the system does not produce any topological transformations. It was also found out that the beam-transferred vortex proves to be structurally stable towards axially symmetric perturbations on the circular diaphragm.

REFERENCES


